Research Article

Structural Analysis Of Soft Influence Sets Via Bipartite Graphs

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ABSTRACT

The concept of soft sets was introduced by Dmitri Molodtsov as a mathematical tool for handling uncertainty and imprecise information, and has since been widely applied to decision-making problems involving vague or ambiguous data. In this paper, we introduce a new class of soft sets, namely soft generalized** ψ -closed sets (briefly, $\mathcal{Sg}^{**}\psi$ -closed sets) and soft generalized** ψ -open sets (briefly, $\mathcal{Sg}^{**}\psi$ -open sets), in the framework of soft topological spaces. We examine the relationships between $\mathcal{Sg}^{**}\psi$ -closed sets and several existing classes of soft closed sets, and establish some of their fundamental properties. Furthermore, we propose a novel approach for representing soft sets by means of bipartite graphs, and show that various operations on soft sets—such as union, intersection, and logical operations—can be modeled and visualized through bipartite graph structures. And we introduce a bipartite graph representation of $\mathcal{Sg}^{**}\psi$ -closed sets and to investigate their associated operations within this graphical framework.

Keywords: $\mathcal{Sg}^{**}\psi$ -closed set, $\mathcal{Sg}^{**}\psi$ -open set, $\mathcal{Sg}^{**}\psi$ -closure, $\mathcal{Sg}^{**}\psi$ -interior, $\mathcal{Sg}^{**}\psi$ -neighborhood, $\mathcal{Sg}^{**}\psi$ –closed bipartite graph, $\mathcal{Sg}^{**}\psi$ – closed complete bipartite graph.

I - INTRODUCTION

Soft set theory, introduced by D. Molodtsov in 1999[14], provides an effective mathematical framework for addressing uncertainties through parameterized structures. The concept of soft topological spaces was later established by Muhammad Shabir and Munazza Naz in 2011[11], which laid the foundation for studying topological properties within the framework of soft sets. Subsequent developments by Naim Çağman[2] and collaborators further enriched the theory by extending definitions and exploring structural properties of soft sets. In classical topology, Generalized closed sets were introduced by Norman Levine in 1970[9], which motivated many researchers to investigate various generalized forms of closed and open sets. Building upon this concept, V. Kannan defined soft generalized closed and soft generalized open sets in soft topological spaces[8] . Earlier studies by M. K. R. S. Veerakumar[24] examined sets lying between closed sets and generalized closed sets. Later, the notion of $g*$-closed sets in topological spaces was proposed by M. Pauline Mary Helen, Veronica Vijayan, and Ponnuthai Selvarani[16]. Further extensions were introduced in the context of soft topology, where A. Devika[3] defined soft $g*$-closed sets. Subsequently, A. Kalavathi and G. Sai Sundara Krishnan[7] introduced both soft $g*$-closed and soft $g*$-open sets in soft topological spaces. In addition, M. A. Abd Allah and A. S. Nawar[1] investigated ψ^{*}-closed sets in fuzzy topological spaces, while more recently N. Gomathi and T. Indira[5] proposed soft $g{*}{*}$-closed sets. The fundamental operations of soft sets, including union, intersection, AND-operation, and OR-operation, were formulated by P. K. Maji[10] and co-authors, which significantly strengthened the theoretical framework of soft set theory. Parallel to these developments, graph theory has evolved as an essential tool for modeling relationships and structures. In 2014[23], Rajesh K. Thumbakara and Boben George proposed the concept of soft graphs, extending soft set theory to graph structures. This idea was further developed by Sumit Mohinta and T. K. Samanta[13] through the introduction of fuzzy soft graphs, which combine the characteristics of fuzzy sets and soft sets in graph models. Additionally, Faiz farid[4] and co-researchers proposed new approaches for representing graphs using adjacency relations within the soft set framework. The motivation for introducing soft generalized** Ψ-closed sets arises from the need to generalize existing soft closed structures to better model uncertainty in parameterized systems. These concepts can be applied in decision-making problems, data classification, and network modeling. The use of bipartite graphs provides a visual and computational framework for representing soft topological relationships

II-PRELIMINARIES

This section introduces essential definitions that are useful for understanding the ideas discussed in this paper and for developing the results presented in this study.

Definition : 2.1 [2]

Let U be a universe and E be a set of parameters and let $A \subseteq E$. A soft set $\mathcal{F}_A$ over U is defined as $\mathcal{F}_A = \{(x, f_A(x)) : x \in E\}$ where $f_A : E \rightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here the value of $f_A(x)$ may be aribitary. Some of them may be empty some may have non-empty intersection.

Note that the set of all soft sets with the parameter set E over U will be denoted by $S(U)$.

Definition : 2.2 [2]

Let $\mathcal{F}_A \in S(U)$. If $f_A(x) = \emptyset$ for all $x \in A$ then F_A is called an empty soft set, denoted by $F_\emptyset$.

Definition: 2.3 [2]

Let $\mathcal{F}_A \in S(U)$. If $\mathcal{F}_A(x) = U$ for all $x \in A$ then$\mathcal{F}_A$ is called a A-universal soft set, denoted by $F_{\tilde{A}}$. If $A = E$, then the A-universal soft set is called universal soft set denoted by $F_{\tilde{E}}$.

Definition: 2.4 [2]

Let $\mathcal{F}_A, \mathcal{F}_B \in S(U)$. Then soft union$\mathcal{F}_A \tilde{\cup} \mathcal{F}_B$, Soft intersection $\mathcal{F}_A \tilde{\cap} \mathcal{F}_B$, and soft difference $\mathcal{F}_A \backslash \mathcal{F}_B$ of $\mathcal{F}_A$ and $\mathcal{F}_A$ are defined by respectively.$f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x)$, $f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$, $f_{A\tilde{\backslash}B}(x) = f_A(x) \backslash f_B(x)$,and the soft complement $F^{\tilde{c}}{}_A$ of $\mathcal{F}_A$ is defined by $f^{\tilde{c}}{}_A(x) = f^c{}_A$ where $f^c{}_A(x)$ is complement of the set $f_A(x)$, that is $f^c{}_A(x) = U \backslash f_A(x)$ for all $x \in E$.

Definition: 2.5 [2]

Let $\mathcal{F}_A \in S(U)$. The relative complement of $\mathcal{F}_A$ is denoted by F'_A and is defined by $(\mathcal{F}_A)' = \mathcal{F}_A{}')$ where $F'_A : A \rightarrow P(U)$ is a mapping given by $F'_\propto = U \backslash F_\propto$ for all $\propto \in A$.

Definition: 2.6

Let $\mathcal{F}_A \in S(U)$. A soft topology on $\mathcal{F}_A$ denoted by $\tilde{\tau}$, is a collection of soft subsets of $\mathcal{F}_A$ having following conditions .

(i) $\mathcal{F}_A, \mathcal{F}_\varphi \in \tilde{\tau}$.

(ii) The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

(iii) The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

Then the pair $(\mathcal{F}_A, \tilde{\tau})$ is called a soft topological space.

Definition: 2.7

Let $(\mathcal{F}_A, \tilde{\tau})$ be a soft topological space, then every element of $\tilde{\tau}$ is called a soft open sets in $\tilde{\tau}$.

Definition: 2.8

Let $(\mathcal{F}_A, \tilde{\tau})$ be a soft topological space. A soft set F_A is said to be a soft closed set, if its relative complement F'_A belongs to $\tilde{\tau}$.

Definition: 2.9

Let $(\mathcal{F}_A, \tilde{\tau})$ be a soft topological space, then soft interior of soft set $\mathcal{F}_A$ is defined as the union of all soft open sets contained in $\mathcal{F}_A$. It is denoted by $int(\mathcal{F}_A)$.

Definition : 2.10

Let $(\mathcal{F}_A, \tilde{\tau})$ be a soft topological space, then soft closure of soft set $\mathcal{F}_A$ is defined as the intersection of all soft closed super sets containing in $\mathcal{F}_A$. It is denoted by $cl(\mathcal{F}_A)$.

Definition : 2.11 [1,3,5,8,18,20]

Let $(\mathcal{F}_A, \tilde{\tau})$ be a soft topological space, a soft set $\mathcal{F}_A$ over U is called a

1) Soft semi open set if $\mathcal{F}_A \subseteq cl\big(int(\mathcal{F}_A)\big)$ and a semi closed set if $int\big(cl(\mathcal{F}_A)\big) \subseteq \mathcal{F}_A$.
2) Soft pre open set if $\mathcal{F}_A \subseteq int\big(cl(\mathcal{F}_A)\big)$ and a pre closed set if $cl\big(int(\mathcal{F}_A)\big) \subseteq \mathcal{F}_A$.
3) Soft α-open set if $\mathcal{F}_A \subseteq int(cl(int(\mathcal{F}_A)))$ and a α-closed set if $(int(cl(\mathcal{F}_A))) \subseteq \mathcal{F}_A$.
4) Soft β-open set if $\mathcal{F}_A \subseteq cl(int(cl(\mathcal{F}_A)))$ and a β-closed set if $int(cl(int(\mathcal{F}_A))) \subseteq \mathcal{F}_A$
5) Soft regular open set if $\mathcal{F}_A = int(cl(\mathcal{F}_A))$ and a regular closed set if $\mathcal{F}_A = cl(int(\mathcal{F}_A))$
6) Soft generalized closed (briefly soft g-closed) set if $cl(\mathcal{F}_A) \subseteq U_A$, whenever $\mathcal{F}_A \subseteq U_A$ and U_A is soft open in $(\mathcal{F}_A, \tilde{\tau})$.
7) Soft semi generalized closed (briefly soft sg-closed) set if $scl(\mathcal{F}_A) \subseteq U_A$, whenever $\mathcal{F}_A \subseteq U_A$ and U_A is soft semi open in $(\mathcal{F}_A, \tilde{\tau})$
8) Soft generalized * closed (briefly soft g*-closed) set if $cl(\mathcal{F}_A) \subseteq U_A$, whenever $\mathcal{F}_A \subseteq U_A$ and U_A is soft g-open in $(\mathcal{F}_A, \tilde{\tau})$.

9) Soft α-generalized closed (briefly α soft g-closed) set if α $cl(F_A) \subseteq U_A$, whenever $F_A \subseteq U_A$ and U_A is soft open in $(\mathcal{F}_A, \tilde{\tau})$.

10) Soft generalized**-closed (briefly soft g**-closed) set if $cl(\mathcal{F}_A) \subseteq U_A$, whenever $\mathcal{F}_A \subseteq U_A$ and U_A is soft g*-open in $(\mathcal{F}_A, \tilde{\tau})$.

11) Soft ψ -closed (briefly soft ψ -closed) set if $scl(\mathcal{F}_A) \subseteq U_A$, whenever $\mathcal{F}_A \subseteq U_A$ and U_A is soft generalized open in $(\mathcal{F}_A, \tilde{\tau})$.

12) Soft α ψ -closed (briefly soft α ψ -closed) set $\psi cl(A) \subseteq U$ if, whenever $\mathcal{F}_A \subseteq U_A$ and U_A is soft α open in $(\mathcal{F}_A, \tilde{\tau})$.

13) Soft ψ g-closed (briefly soft ψ g-closed) set $\psi cl(A) \subseteq U$ if, whenever $\mathcal{F}_A \subseteq U_A$ and U_A is soft open in $(\mathcal{F}_A, \tilde{\tau})$.

14) Soft ψ *α-closed (briefly soft ψ *α -closed) set $\alpha cl(A) \subseteq U$ if, whenever $\mathcal{F}_A \subseteq U_A$ and U_A is soft ψ g open in $(\mathcal{F}_A, \tilde{\tau})$.

15) Soft g* ψ -closed (briefly soft g* ψ -closed) set $\psi cl(A) \subseteq U$ if, whenever $\mathcal{F}_A \subseteq U_A$ and U_A is soft g-open in $(\mathcal{F}_A, \tilde{\tau})$.

16) Soft ψ *g*-closed (briefly soft ψ *g*-closed) set $\psi cl(A) \subseteq U$ if, whenever $\mathcal{F}_A \subseteq U_A$ and U_A is soft ψ g- open in $(\mathcal{F}_A, \tilde{\tau})$.

The relative complements of the above soft closed sets are called Soft open sets respectively.

III – SOFT GENERALIZED**Ψ- CLOSED SETS IN SOFT TOPOLOGICAL SPACES

In this section, we introduce the notion of soft generalized **Ψ-closed sets and examine their relationships with the existing classes of soft closed sets in a more comprehensive manner.

Definition:3.1

A soft set $\mathcal{F}_A$ is called a soft generalized **Ψ**-** closed (briefly $\mathcal{S}g^{**}\psi$ - closed) set if $\psi cl(\mathcal{F}_A) \subseteq U_A$, whenever $\mathcal{F}_A \subseteq U_A$ and U_A is soft g*- open in $(\mathcal{F}_A, \tilde{\tau})$.

Example :3.2

Let U={ α, β, γ}, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\} \subseteq E$, $\mathcal{F}_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\beta, \gamma\})\}$
$\mathcal{F}_{A_1} = \{(J_1, \{\alpha\})\}$, $\mathcal{F}_{A_2} = \{(J_1, \{\beta\})\}$, $\mathcal{F}_{A_3} = \{(J_1, \{\alpha, \beta\})\}$, $\mathcal{F}_{A_4} = \{(J_2, \{\beta\})\}$, $\mathcal{F}_{A_5} = \{(J_2, \{\gamma\})\}$, $\mathcal{F}_{A_6} = \{(J_2, \{\beta, \gamma\})\}$, $\mathcal{F}_{A_7} = \{(J_1, \{\alpha\}), (J_2, \{\beta\})\}$, $\mathcal{F}_{A_8} = \{(J_1, \{\alpha\}), (J_2, \{\gamma\})\}$, $\mathcal{F}_{A_9} = \{(J_1, \{\alpha\}), (J_2, \{\beta, \gamma\})\}$, $\mathcal{F}_{A_{10}} = \{(J_1, \{\beta\}), (J_2, \{\beta\})\}$, $\mathcal{F}_{A_{11}} = \{(J_1, \{\beta\}), (J_2, \{\gamma\})\}$, $\mathcal{F}_{A_{12}} = \{(J_1, \{\beta\}), (J_2, \{\beta, \gamma\})\}$, $\mathcal{F}_{A_{13}} = \{(J_1, \{\alpha, \beta\}), (J_2, \{\beta\})\}$, $\mathcal{F}_{A_{14}} = \{(J_1, \{\alpha, \beta\}), (J_2, \{\gamma\})\}$, $\mathcal{F}_{A_{15}} = \mathcal{F}_A$, $\mathcal{F}_{A_{16}} = \mathcal{F}_{\varphi}$. Soft open sets $(\tilde{\tau}) = \{\mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_2}, \mathcal{F}_{A_3}, \mathcal{F}_{A_{11}}, \mathcal{F}_{A_{12}}, \mathcal{F}_{A_{14}}\}$, Soft closed sets $(\tilde{\tau})^C = \{\mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_9}, \mathcal{F}_{A_6}, \mathcal{F}_{A_7}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}\}$. Then $(\mathcal{F}_A, \tilde{\tau})$ is a soft topological space , and $\mathcal{S}g^{**}\psi$ - closed sets= $\{\mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}, \mathcal{F}_{A_5}, \mathcal{F}_{A_6}, \mathcal{F}_{A_7}, \mathcal{F}_{A_8}, \mathcal{F}_{A_9}\}$.

Theorem: 3.3

(i) Every soft closed set is also a $\mathcal{S}g^{**}\psi$ - closed set.

(ii) Every soft g**- closed set is also a $\mathcal{S}g^{**}\psi$ - closed set.

Proof:

(i) Let $\mathcal{F}_A$ be a soft closed set in and U_A be a soft g*-open in $(\mathcal{F}_A, \tilde{\tau})$ such that $\mathcal{F}_A \subseteq U_A$. We know that "Every soft closed set is soft ψ - closed set". $\Rightarrow cl(\mathcal{F}_A) \subseteq \psi cl(\mathcal{F}_A) \Rightarrow \psi cl(\mathcal{F}_A) = \mathcal{F}_A \subseteq U_A$. $\Rightarrow \psi cl(\mathcal{F}_A) \subseteq U_A$ and U_A is soft g*-open set in $(\mathcal{F}_A, \tilde{\tau})$. Hence $\mathcal{F}_A$ *is* $\mathcal{S}g^{**}\psi$ - closed set.

(ii) The result can be established using arguments similar to those used in the proof of part (i). Hence, the proof is omitted for brevity.

The converse of the above theorems does not hold in general, as demonstrated by the following counterexample.

Example: 3.4

(i) Consider the Example 3.2, U={ α, β, γ}, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\} \subseteq E$, $\mathcal{F}_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\beta, \gamma\})\}$. Soft open sets $(\tilde{\tau}) = \{\mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_2}, \mathcal{F}_{A_3}, \mathcal{F}_{A_{11}}, \mathcal{F}_{A_{12}}, \mathcal{F}_{A_{14}}\}$, Soft closed sets $(\tilde{\tau})^C =$

$\{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_9}, \mathcal{F}_{A_6}, \mathcal{F}_{A_7}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}\}$. Then $(\mathcal{F}_A, \tilde{\tau})$ is a soft topological space , and $\mathcal{S}g^{**}\psi$ - closed sets= $\{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}, \mathcal{F}_{A_5}, \mathcal{F}_{A_6}, \mathcal{F}_{A_7}, \mathcal{F}_{A_8}, \mathcal{F}_{A_9}\}$. Consider $\mathcal{F}_{A_8} = \{(J_1, \{\alpha\}), (J_2, \{\gamma\})\}$ and $\mathcal{F}_{A_5} = \{(J_2, \{\gamma\})\}$ are $\mathcal{S}g^{**}\psi$ - closed sets but not soft closed sets.

(ii) Let U=$\{\alpha, \beta, \gamma, \delta\}$, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\} \subseteq E$. $\mathcal{F}_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\delta\})\}$, $\mathcal{F}_{A_1} = \{(J_1, \{\alpha\})\}$, $\mathcal{F}_{A_2} = \{(J_1, \{\beta\})\}$, $F_{A_3} = \{(J_1, \{\alpha, \beta\})\}$, $\mathcal{F}_{A_4} = \{(J_1, \{\alpha\}), (J_2, \{\delta\})\}$, $\mathcal{F}_{A_5} = \{(J_1, \{\beta\}), (J_2, \{\delta\})\}$, $\mathcal{F}_{A_6} = \{(J_2, \{\delta\})\}$, $\mathcal{F}_{A_7} = F_{A,} \mathcal{F}_{A_8} = \mathcal{F}_{\varphi}$. Soft open sets $(\tilde{\tau}) = \{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_5}, \mathcal{F}_{A_2}\}$, Soft closed sets $(\tilde{\tau})^C = \{ \mathcal{F}_A, F_{\emptyset}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}\}$. Then $(\mathcal{F}_A, \tilde{\tau})$ is a soft topological space. $\mathcal{S}g^{**}\psi$ - closed sets= $\{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_3}, \mathcal{F}_{A_4}, \mathcal{F}_{A_6} \}$. Here $\mathcal{F}_{A_6} = \{(e_2, \{\delta\})\}$ is $\mathcal{S}g^{**}\psi$ - closed set but not soft generalized**-closed set.

Theorem: 3.5

Every soft α - closed set is also a $\mathcal{S}g^{**}\psi$ - closed set.

Proof:

Let $\mathcal{F}_A$ be a soft α- closed set in and U_A be a soft g*-open in $(\mathcal{F}_A, \tilde{\tau})$ such that $\mathcal{F}_A \subseteq U_A$. We know that "Every soft α-closed set is soft ψ - closed set". $\Rightarrow \alpha cl(\mathcal{F}_A) \subseteq \psi cl(\mathcal{F}_A) \Rightarrow \psi cl(\mathcal{F}_A) = \mathcal{F}_A \subseteq U_A$. $\Rightarrow \psi cl(\mathcal{F}_A) \subseteq U_A$ and U_A is soft g*-open set in $(\mathcal{F}_A, \tilde{\tau})$. Hence $\mathcal{F}_A$ *is* $\mathcal{S}g^{**}\psi$ - closed set.

The converse of the above theorems does not hold in general, as demonstrated by the following counterexample

Example: 3.6

Consider the example 3.4 (ii) Let U=$\{\alpha, \beta, \gamma, \delta\}$, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\} \subseteq E$. $\mathcal{F}_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\delta\}$. Soft open sets $(\tilde{\tau}) = \{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_5}, \mathcal{F}_{A_2}\}$, Soft closed sets $(\tilde{\tau})^C = \{ \mathcal{F}_A, F_{\emptyset}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}\}$. Then $(\mathcal{F}_A, \tilde{\tau})$ is a soft topological space. $\mathcal{S}g^{**}\psi$ - closed sets $= \{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_3}, \mathcal{F}_{A_4}, \mathcal{F}_{A_6} \}$. soft α -closed sets $= \{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}, \mathcal{F}_{A_6} \}$. Here $\mathcal{F}_{A_3} = \{(J_1, \{\alpha, \beta\})\}$ is $\mathcal{S}g^{**}\psi$ - closed set but it is not soft α-closed set.

Theorem: 3.7

Every soft ψ - closed set is also a $\mathcal{S}g^{**}\psi$ - closed set.

Proof:

Let $\mathcal{F}_A$ be a soft ψ - closed set in and U_A be a soft g*-open in $(\mathcal{F}_A, \tilde{\tau})$ such that $\mathcal{F}_A \subseteq U_A$. We know that "Every soft closed set is soft ψ - closed set". $\Rightarrow cl(\mathcal{F}_A) \subseteq \psi cl(\mathcal{F}_A)$ and "Every soft g-open set is soft g*-open set $\Rightarrow \psi cl(\mathcal{F}_A) = \mathcal{F}_A \subseteq U_A$. $\Rightarrow \psi cl(\mathcal{F}_A) \subseteq U_A$ and U_A is soft g*-open set in $(\mathcal{F}_A, \tilde{\tau})$. Hence $\mathcal{F}_A$ *is* $\mathcal{S}g^{**}\psi$ - closed set.

The converse of the above theorems does not hold in general, as demonstrated by the following counterexample

Example: 3.8

Consider the example 3.4 (ii) Let U=$\{\alpha, \beta, \gamma, \delta\}$, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\} \subseteq E$. $\mathcal{F}_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\delta\}$. Soft open sets $(\tilde{\tau}) = \{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_5}, \mathcal{F}_{A_2}\}$, Soft closed sets $(\tilde{\tau})^C = \{ \mathcal{F}_A, F_{\emptyset}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}\}$. Then $(\mathcal{F}_A, \tilde{\tau})$ is a soft topological space. $\mathcal{S}g^{**}\psi$ - closed sets $= \{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_3}, \mathcal{F}_{A_4}, \mathcal{F}_{A_6} \}$. soft ψ -closed sets $= \{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}, \mathcal{F}_{A_6} \}$. Here $F_{A_3} = \{(e_1, \{\alpha, \beta\})\}$ is $\mathcal{S}g^{**}\psi$ - closed set but it is not soft ψ-closed set.

Theorem: 3.9

Every soft semi generalized closed set is also a $\mathcal{S}g^{**}\psi$ - closed set

Proof:

Let $\mathcal{F}_A$ be a soft ψ - closed set in and U_A be a soft g*-open in $(\mathcal{F}_A, \tilde{\tau})$ such that $\mathcal{F}_A \subseteq U_A$. We know that "Every soft semi open set is soft g*- open set". $\Rightarrow cl(\mathcal{F}_A) \subseteq \psi cl(\mathcal{F}_A) \Rightarrow \psi cl(\mathcal{F}_A) = \mathcal{F}_A \subseteq U_A$. $\Rightarrow \psi cl(\mathcal{F}_A) \subseteq U_A$ and U_A is soft g*-open set in $(\mathcal{F}_A, \tilde{\tau})$. Hence $\mathcal{F}_A$ *is* $\mathcal{S}g^{**}\psi$ - closed set.

The converse of the above theorems does not hold in general, as demonstrated by the following counterexample

Example 3.10

Consider the example 3.4 (ii). Let U=$\{\alpha, \beta, \gamma, \delta\}$, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\} \subseteq E$. $\mathcal{F}_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\delta\}$. Soft open sets $(\tilde{\tau}) = \{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_5}, \mathcal{F}_{A_2}\}$, Soft closed sets $(\tilde{\tau})^C = \{ \mathcal{F}_A, F_{\emptyset}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}\}$. Then $(\mathcal{F}_A, \tilde{\tau})$ is a soft topological space. $\mathcal{S}g^{**}\psi$ - closed sets $= \{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_3}, \mathcal{F}_{A_4}, \mathcal{F}_{A_6} \}$. soft semi generalized-closed sets $= \{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}, \mathcal{F}_{A_6} \}$. Here $F_{A_3} = \{(e_1, \{\alpha, \beta\})\}$ is $\mathcal{S}g^{**}\psi$ - closed set but it is not soft semi generalized closed set.

Remark: 3.11

I. Soft generalized semi-closed sets and $\mathcal{S}g^{**}\psi$ are mutually interdependent.
II. Soft ψ generalized-closed sets and $\mathcal{S}g^{**}\psi$ are mutually interdependent.
III. Soft g* ψ-closed sets and $\mathcal{S}g^{**}\psi$ are mutually interdependent.
IV. soft $\psi\hat{g}$ -closed sets and $\mathcal{S}g^{**}\psi$ are mutually interdependent.

The mutual interdependence of the aforementioned sets is substantiated by the following example.

Example: 3.12

- Consider the Example 3.4 (ii). Here soft generalized semi closed sets and $\mathcal{S}g^{**}\psi$ -closed sets are $\{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_3}, \mathcal{F}_{A_4}, \mathcal{F}_{A_6} \}$.
- Let $U = \{\alpha, \beta, \gamma, \delta\}$, $E = \{J_1, J_2, J_3\}$ and $A = \{ J_2, J_3 \} \subseteq E$, $\mathcal{F}_A = \{(J_2, \{\alpha\}), (J_3, \{\beta, \gamma\})\}$,

$\mathcal{F}_{A_1} = \{(J_2, \{\alpha\})\}$, $\mathcal{F}_{A_2} = \{(J_2, \{\alpha\}), (J_3, \{\beta\})\}$, $\mathcal{F}_{A_3} = \{(J_2, \{\alpha\}), (J_3, \{\gamma\})\}$, $\mathcal{F}_{A_4} = \mathcal{F}_A$
$\mathcal{F}_{A_5} = \{(J_3, \{\beta\})\}$, $\mathcal{F}_{A_6} = \{(J_3, \{\gamma\})\}$, $\mathcal{F}_{A_7} = \{(J_3, \{\beta, \gamma\})\}$, $\mathcal{F}_{A_8} = \mathcal{F}_{\varphi}$. Soft open sets $(\tilde{\tau}) = \{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_2}, \mathcal{F}_{A_7}\}$, Soft closed sets $(\tilde{\tau})^C = \{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_3}\}$. Then $(\mathcal{F}_A, \tilde{\tau})$ is a soft topological space. $\mathcal{S}g^{**}\psi$ -closed sets = soft ψg-closed sets = soft g* ψ-closed sets = soft $\psi\hat{g}$ -closed sets = $\{ \mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_3}, \mathcal{F}_{A_6}\}$.

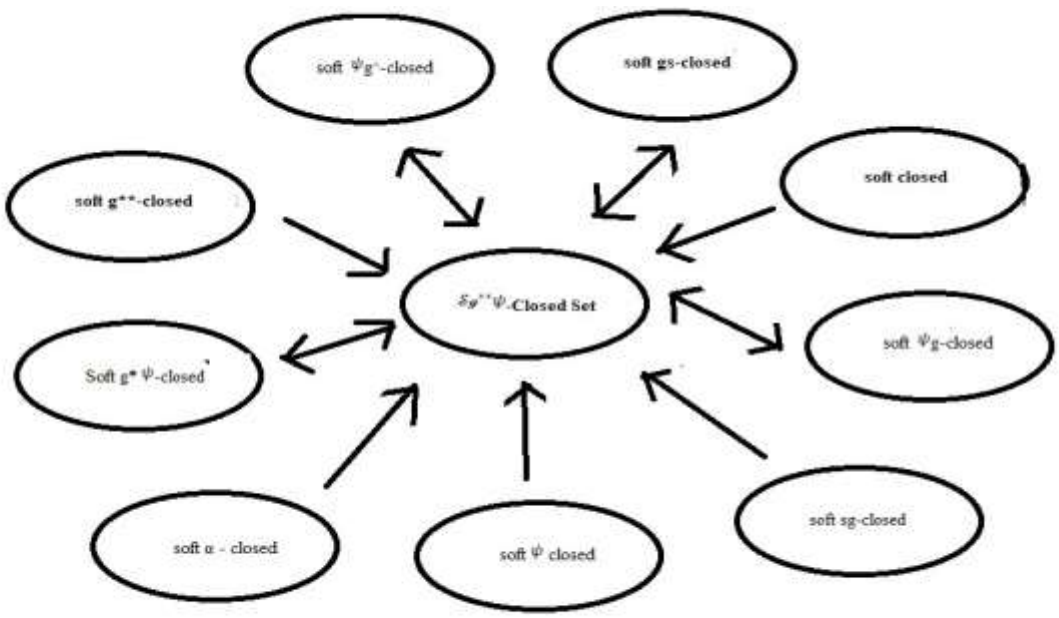

Figure-1 The diagram depicts the implication structure connecting the existing classes of soft closed sets with the newly proposed class of $\mathcal{S}g^{**}\psi$ -closed sets.

Theorem: 3.13

If $\mathcal{F}_A$ is $\mathcal{S}g^{**}\psi$ -closed set in $(\mathcal{F}_A, \tilde{\tau})$ and $\mathcal{F}_A \subseteq \mathcal{G}_A \subseteq \psi cl(\mathcal{F}_A)$ then $\mathcal{G}_A$ is $\mathcal{S}g^{**}\psi$ -closed set.

Proof:

Suppose that $\mathcal{F}_A$ is $\mathcal{S}g^{**}\psi$ -closed set in $(\mathcal{F}_A, \tilde{\tau})$ and $\mathcal{F}_A \subseteq \mathcal{G}_A \subseteq \psi cl(F_A)$. Let $\mathcal{G}_A \subseteq U_A$ and U_A is soft g*-open set in $(\mathcal{F}_A, \tilde{\tau})$. Since $\mathcal{F}_A \subseteq \mathcal{G}_A$ and $\mathcal{G}_A \subseteq U_A \Rightarrow \mathcal{F}_A \subseteq U_A$. Hence $\psi cl(\mathcal{F}_A) \subseteq (U_A)$ [Since $\mathcal{F}_A$ is $\mathcal{S}g^{**}\psi$ -closed]. $\Rightarrow$ $\mathcal{G}_A \subseteq \psi cl(\mathcal{F}_A)$. $\Rightarrow$ $\mathcal{G}_A \subseteq U_A$ and U_A is soft g*-open set in $(\mathcal{F}_A, \tilde{\tau})$. Therefore $\mathcal{G}_A$ is $\mathcal{S}g^{**}\psi$ -closed set.

Theorem: 3.14

The finite intersection of $\mathcal{S}g^{**}\psi$ -closed sets is also a $\mathcal{S}g^{**}\psi$ -closed set.

Proof:

Suppose that $\mathcal{F}_A$ and $\mathcal{G}_A$ are two arbitrary $\mathcal{S}g^{**}\psi$ -closed sets . Let $\mathcal{F}_A \cap \mathcal{G}_A \sqsubseteq U_A$ is soft g*-open set. $\Rightarrow \mathcal{F}_A \sqsubseteq U_A$ and $\mathcal{G}_A \sqsubseteq U_A$. We know that U_A is soft g*-open set in $(\mathcal{F}_A, \tilde{\tau})$ and $\mathcal{F}_A$ and $\mathcal{G}_A$ are $\mathcal{S}g^{**}\psi$ - closed sets $\Rightarrow \psi cl(\mathcal{F}_A) \sqsubseteq U_A$ and $\psi cl(\mathcal{G}_A) \sqsubseteq U_A$. Therefore $\psi cl\{(\mathcal{F}_A \cap \mathcal{G}_A)\} = \psi cl(\mathcal{F}_A) \cap \psi cl(\mathcal{G}_A) \sqsubseteq U_A$. The following example illustrates the above result.

Example: 3.15

Consider the example 3.2, Let U={ $\alpha, \beta, \gamma\}$, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\} \sqsubseteq E$, $\mathcal{F}_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\beta, \gamma\})\}$.
Soft open sets $(\tilde{\tau}) = \{\mathcal{F}_A, \mathcal{F}_\varphi, \mathcal{F}_{A_2}, \mathcal{F}_{A_3}, \mathcal{F}_{A_{11}}, \mathcal{F}_{A_{12}}, \mathcal{F}_{A_{14}}\}$, Soft closed sets $(\tilde{\tau})^C =$
$\{\mathcal{F}_A, \mathcal{F}_\varphi, \mathcal{F}_{A_9}, \mathcal{F}_{A_6}, \mathcal{F}_{A_7}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}$and $\mathcal{S}g^{**}\psi$ - closed sets= $\{\mathcal{F}_A, \mathcal{F}_\varphi, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}, \mathcal{F}_{A_5}, \mathcal{F}_{A_6}, \mathcal{F}_{A_7}, \mathcal{F}_{A_8}, \mathcal{F}_{A_9}\}$. $\mathcal{F}_{A_1} \cap \mathcal{F}_{A_7} = \mathcal{F}_{A_1}$, $\mathcal{F}_{A_7} \cap \mathcal{F}_{A_8} = \mathcal{F}_{A_1}$, $\mathcal{F}_{A_8} \cap \mathcal{F}_{A_9} = \mathcal{F}_{A_8}$.

Theorem: 3.16

If a set $\mathcal{F}_A$ is $\mathcal{S}g^{**}\psi$ -closed set in $(\mathcal{F}_A, \tilde{\tau})$ if and only if $\psi cl(\mathcal{F}_A)\backslash(\mathcal{F}_A)$ contains only null soft closed set.

Proof:

Suppose that $\mathcal{F}_A$ is $\mathcal{S}g^{**}\psi$-closed set in $(\mathcal{F}_A, \tilde{\tau})$. Let $\mathcal{G}_A$ be soft closed set and $\mathcal{G}_A \sqsubseteq \psi cl(\mathcal{F}_A)\backslash(\mathcal{F}_A)$. Since $\mathcal{G}_A$ is soft closed and its relative complement $\mathcal{G}'_A$ is soft open. Now $\mathcal{G}_A \sqsubseteq \psi cl(\mathcal{F}_A)\backslash(\mathcal{F}_A) \Rightarrow \mathcal{G}_A \sqsubseteq \psi cl(\mathcal{F}_A)$ and $\mathcal{G}_A \not\sqsubset \mathcal{F}_A$. $\Rightarrow$ $\mathcal{G}_A \sqsubseteq \psi cl(\mathcal{F}_A)$ and $\mathcal{G}_A \sqsubseteq \mathcal{F}'_A$. Hence $\mathcal{F}_A \sqsubseteq \mathcal{G}'_A$. Consequently $\psi cl(\mathcal{F}_A) \sqsubseteq \mathcal{G}'_A$. [Since $\mathcal{F}_A$ is $\mathcal{S}g^{**}\psi$ -closed set]. Therefore $\mathcal{G}_A \sqsubseteq \psi cl(\mathcal{F}'_A)$. $\Rightarrow \mathcal{G}_A = \psi\ cl(\mathcal{F}_A) \bigcap (\mathcal{F}'_A) = \emptyset$. $\Rightarrow \mathcal{G}_A = \emptyset$. Therefore $\psi cl(\mathcal{F}_A)\backslash(\mathcal{F}_A)$ contains only null soft closed set.

Conversely, $\psi cl(\mathcal{F}_A)\backslash(\mathcal{F}_A)$ contains only null soft closed set.

Since $\psi cl(\mathcal{F}_A)\backslash(\mathcal{F}_A) = \emptyset$. $\Rightarrow \psi cl(\mathcal{F}_A) = \mathcal{F}_A \Rightarrow \mathcal{F}_A$ is soft closed set. We know that every soft closed set is $\mathcal{S}g^{**}\psi$ - closed set . Therefore $\mathcal{F}_A$ is $\mathcal{S}g^{**}\psi$ - closed set.

Theorem: 3.17

A $\mathcal{S}g^{**}\psi$ - closed set $\mathcal{F}_A$ is soft closed if and only if $\psi cl(\mathcal{F}_A)\backslash(\mathcal{F}_A)$ is soft closed.

Proof:

If $\mathcal{F}_A$ is soft closed. $\Rightarrow cl(\mathcal{F}_A) = F_A$. Therefore $\psi cl(\mathcal{F}_A)\backslash(\mathcal{F}_A) = \emptyset$ and is soft closed set. Conversely, Suppose that $\psi cl(\mathcal{F}_A)\backslash(\mathcal{F}_A)$ is soft closed, Now by theorem 3.9, If a set $\mathcal{F}_A$ is $\mathcal{S}g^{**}\psi$ -closed set in $(\mathcal{F}_A, \tilde{\tau})$ if and only if $\psi cl(\mathcal{F}_A)\backslash(\mathcal{F}_A)$ contains only null soft closed set. Therefore $\mathcal{F}_A$ is soft closed.

IV – SOFT GENERALIZED**Ψ- OPEN SETS IN SOFT TOPOLOGICAL SPACES

In this section, we introduce the concepts of soft generalized **Ψ-open sets and soft generalized **Ψ-neighborhoods, and investigate their fundamental properties.

Definition: 4.1

A soft set $\mathcal{F}_A$ is called a soft generalized** ψ - open (briefly $\mathcal{S}g^{**}\psi$ -open)set in a soft topological space $(\mathcal{F}_A, \tilde{\tau})$, if the relative complement $\mathcal{F}'_A$ is soft g** ψ -closed set in $(F_A, \tilde{\tau})$.

Equivalently, A soft set $\mathcal{F}_A$ *is* $\mathcal{S}g^{**}\psi$, if $U_A \sqsubseteq \psi int(\mathcal{F}_A)$ whenever $U_A \sqsubseteq \mathcal{F}_A$ and U_A is soft g*-closed set.

Example: 4.2

Consider the example 3.2, U={ $\alpha, \beta, \gamma\}$, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\} \sqsubseteq E$, $\mathcal{F}_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\beta, \gamma\})\}$. Soft open sets $(\tilde{\tau}) = \{\mathcal{F}_A, \mathcal{F}_\varphi, \mathcal{F}_{A_2}, \mathcal{F}_{A_3}, \mathcal{F}_{A_{11}}, \mathcal{F}_{A_{12}}, \mathcal{F}_{A_{14}}\}$, Soft closed sets $(\tilde{\tau})^C =$
$\{\mathcal{F}_A, \mathcal{F}_\varphi, \mathcal{F}_{A_9}, \mathcal{F}_{A_6}, \mathcal{F}_{A_7}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}\}$. Then $(\mathcal{F}_A, \tilde{\tau})$ is a soft topological space , and $\mathcal{S}g^{**}\psi$ – open sets=
$\{\mathcal{F}_A, \mathcal{F}_\varphi, \mathcal{F}_{A_2}, \mathcal{F}_{A_3}, \mathcal{F}_{A_5}, \mathcal{F}_{A_{10}}, \mathcal{F}_{A_{11}}, \mathcal{F}_{A_{12}}, \mathcal{F}_{A_{13}}, \mathcal{F}_{A_{14}}\}$.

Theorem: 4.3

Each soft open set is also a $\mathcal{S}g^{**}\psi$ - open set

Proof:

Let $\mathcal{F}_A$ be a soft open set in $(\mathcal{F}_A, \tilde{\tau})$ and let U_A be a soft g*-closed set in $(\mathcal{F}_A, \tilde{\tau})$ such that $\mathcal{F}_A \sqsubseteq U_A$. Since $\mathcal{F}_A$ is soft open set , $int(\mathcal{F}_A) = \mathcal{F}_A$. $\Rightarrow int(\mathcal{F}_A) = \mathcal{F}_A \sqsubseteq U_A$ and since "Every soft open set is soft ψ-open set". Therefore $int(\mathcal{F}_A) \sqsubseteq \psi int(\mathcal{F}_A) \sqsubseteq U_A$. $\Rightarrow \psi int(\mathcal{F}_A) \sqsubseteq U_A$ and U_A is soft g*- closed set in $(\mathcal{F}_A, \tilde{\tau})$. Hence $\mathcal{F}_A$ is soft g**- open set.

The converse of the above theorem need not be valid, which can be seen from the following example.

Remark: 4.4

Consider the example 3.2, U={ α,β,γ}, $E=\{J_1,J_2\ ,J_3\}$ and $A=\{J_1,J_2\}\subseteq E$, $\mathcal{F}_A=\{(J_1,\{\alpha,\beta\}),(J_2,\{\beta,\gamma\})\}$. Soft open sets $(\tilde{\tau})\ =\ \{\mathcal{F}_A,\mathcal{F}_\varphi,\mathcal{F}_{A_2},\mathcal{F}_{A_3},\mathcal{F}_{A_{11}},\mathcal{F}_{A_{12}},\mathcal{F}_{A_{14}}\}$, Soft closed sets $(\tilde{\tau})^C\ =\{\mathcal{F}_A,\mathcal{F}_\varphi,\mathcal{F}_{A_9},\mathcal{F}_{A_6},\mathcal{F}_{A_7},\mathcal{F}_{A_1},\mathcal{F}_{A_4}\}$. Then $(\mathcal{F}_A,\tilde{\tau})$ is a soft topological space , and $\mathcal{S}g^{**}\psi$ –open sets = $\{\mathcal{F}_A,\mathcal{F}_\varphi,\mathcal{F}_{A_2},\mathcal{F}_{A_3},\mathcal{F}_{A_5},\ \mathcal{F}_{A_{10}},\mathcal{F}_{A_{11}},\mathcal{F}_{A_{12}},\mathcal{F}_{A_{13}},\mathcal{F}_{A_{14}}\}$. Here $\mathcal{F}_{A_5}$, $\mathcal{F}_{A_{10}},\mathcal{F}_{A_{13}}$ are $\mathcal{S}g^{**}\psi$- open sets but not soft open sets.

Theorem: 4.5

If $\mathcal{F}_A$ is $\mathcal{S}g^{**}\psi$ -open set in $(\mathcal{F}_A,\tilde{\tau})$ and $\psi int(\mathcal{F}_A)\subseteq\mathcal{G}_A\subseteq\mathcal{F}_A$, then $\mathcal{G}_A$ is $\mathcal{S}g^{**}\psi$ - open set.

Proof:

Suppose that $\mathcal{F}_A$ is soft $\mathcal{S}g^{**}\psi$ -open set in $(\mathcal{F}_A,\tilde{\tau})$ and $\psi int(\mathcal{F}_A)\subseteq\mathcal{G}_A\subseteq\mathcal{F}_A$. Let $\mathcal{H}_A\subseteq\mathcal{G}_A$ and $\mathcal{H}_A$ is soft g*-closed set in $(\mathcal{F}_A,\tilde{\tau})$. Since $\mathcal{G}_A\subseteq\mathcal{F}_A$ and $\mathcal{H}_A\subseteq\mathcal{G}_A$, we have $\mathcal{H}_A\subseteq\mathcal{F}_A$. Since $\mathcal{F}_A$ is $\mathcal{S}g^{**}\psi$ - open. $\Rightarrow\mathcal{H}_A\subseteq\psi int(\mathcal{F}_A)$, also since $\psi int(\mathcal{F}_A)\subseteq\mathcal{G}_A\Rightarrow\mathcal{H}_A\subseteq\psi int(\mathcal{F}_A)\subseteq\psi int(\mathcal{G}_A)$
$\Rightarrow\mathcal{H}_A\subseteq\psi int(\mathcal{G}_A)$. Therefore $\mathcal{G}_A$ is $\mathcal{S}g^{**}\psi$ -open set.

Theorem: 4.6

The arbitrary union of $\mathcal{S}g^{**}\psi$ -open sets is also a $\mathcal{S}g^{**}\psi$ -open set.

Proof:

Suppose that $\mathcal{F}_A$ and $\mathcal{G}_A$ are $\mathcal{S}g^{**}\psi$ -open sets . Let $U_A\subseteq\mathcal{F}_A\cup\mathcal{G}_A$ and U_A is soft g*-closed set in $(\mathcal{F}_A,\tilde{\tau})$. Since $U_A\subseteq\mathcal{F}_A\cup\mathcal{G}_A$, $\Rightarrow U_A\subseteq\mathcal{F}_A$ and $U_A\subseteq\mathcal{G}_A$. We know that U_A is soft g*-closed set in $(\mathcal{F}_A,\tilde{\tau})$ and $\mathcal{F}_A$ and $\mathcal{G}_A$ are soft g**-open sets $\Rightarrow U_A\subseteq\psi int(\mathcal{F}_A)\ and\ U_A\subseteq\psi int(\mathcal{G}_A)$. Therefore $U_A\subseteq\psi int(\mathcal{F}_A)\cup\psi int(\mathcal{G}_A)$.

Definition: 4.7

Let $(\mathcal{F}_A,\tilde{\tau})$ be a soft topological space, $\mathcal{F}_B\subseteq\mathcal{F}_A$ and $\in\boldsymbol{F}_B$. If there exists a $\mathcal{S}g^{**}\psi$ - open set $\mathcal{F}_C$ such that $\alpha\in\mathcal{F}_C\subseteq\mathcal{F}_B$ then α is called a $\mathcal{S}g^{**}\psi$ - interior point of $\mathcal{F}_B$ and the soft union of all $\mathcal{S}g^{**}\psi$ -interior points of $\mathcal{F}_B$ is denoted by $\mathcal{S}g^{**}\psi-int(\mathcal{F}_B)$.

Definition: 4.8

Let A $(\mathcal{F}_A,\tilde{\tau})$ be a soft topological space, $\mathcal{F}_B\subseteq\mathcal{F}_A$ and $\alpha\in\mathcal{F}_A$. If there exists a soft $\mathcal{S}g^{**}\psi$ - open set $\mathcal{F}_C$ such that $\alpha\in\mathcal{F}_C\subseteq\mathcal{F}_B$ then $\mathcal{F}_B$ is called a $\mathcal{S}g^{**}\psi$ -neighbourhood of α. Set of all $\mathcal{S}g^{**}\psi$ -neighbourhoods of α is denoted by $\mathcal{N}(\alpha)$, is called family of $\mathcal{S}g^{**}\psi$ - neighbourhoods of α that is $\mathcal{N}(\alpha)=\{\mathcal{F}_B:\mathcal{F}_C\in\tilde{\tau}\}$ and $\alpha\in\mathcal{F}_C\subseteq\mathcal{F}_B$. In particular, $V(\alpha)=\{\mathcal{F}_C\in\tilde{\tau}:\alpha\in\mathcal{F}_C\}$.

Example: 4.9

Let U=$\{\alpha,\beta,\gamma,\delta\}$, $E=\{J_1,J_2\ ,J_3\}$ and $A=\{J_1,J_2\}\subseteq E$. $\mathcal{F}_A=\{(J_1,\{\alpha,\beta\}),(J_2,\{\delta\})\}$, $\mathcal{F}_{A_1}=\{(e_1,\{\alpha\})\}$, $\mathcal{F}_{A_2}=\{(e_1,\{\beta\})\}$, $F_{A_3}=\{(e_1,\{\alpha,\beta\})\}$, $\mathcal{F}_{A_4}=\{(e_1,\{\alpha\}),(e_2,\{\delta\})\}$, $\mathcal{F}_{A_5}=\{(e_1,\{\beta\}),(e_2,\{\delta\})\}$, $\mathcal{F}_{A_6}=\{(e_2,\{\delta\})\}$, $\mathcal{F}_{A_7}=F_A$, $\mathcal{F}_{A_8}=\mathcal{F}_\varphi$. Soft open sets $(\tilde{\tau})\ =\{\mathcal{F}_A,\mathcal{F}_\varphi,\mathcal{F}_{A_5},\mathcal{F}_{A_2}\}$, Soft closed sets $(\tilde{\tau})^C\ =\{\mathcal{F}_A,F_\emptyset,\mathcal{F}_{A_1},\mathcal{F}_{A_4}\}$. Then $(\mathcal{F}_A,\tilde{\tau})$ is a soft topological space. $\mathcal{S}g^{**}\psi$ - closed sets= $\{\mathcal{F}_A,\mathcal{F}_\varphi,\mathcal{F}_{A_1},\mathcal{F}_{A_3},\mathcal{F}_{A_4},\mathcal{F}_{A_6}\}$ and $\mathcal{S}g^{**}\psi$ - open sets= $\{\mathcal{F}_A,\mathcal{F}_\varphi,\mathcal{F}_{A_2},\mathcal{F}_{A_5}\}$

Let (i) $\alpha_1=\{(e_1,\{a\})\}$. Then $\mathcal{S}g^{**}\psi$ -neighbourhoods $\tilde{N}(\alpha_1)\ =\ \{\mathcal{F}_A\}$ and $\mathcal{S}g^{**}\psi$ open neighbourhoods $\tilde{V}(\alpha_1)\ =\{\mathcal{F}_A\}$.

(ii) $\alpha_2=\{(e_1,\{b\})\}$. Then $\mathcal{S}g^{**}\psi$ neighbourhoods $\tilde{N}(\alpha_2)\ =\ \{\mathcal{F}_A,\mathcal{F}_{A_3},\mathcal{F}_{A_5}\}$ and $\mathcal{S}g^{**}\psi$ open neighbourhoods $\tilde{V}(\alpha_2)\ =\{\mathcal{F}_A\}$..

(iii) $\alpha_3=\{(e_1,\{a,b\})\}$. Then $\mathcal{S}g^{**}\psi$ neighbourhoods $\tilde{N}(\alpha_3)\ =\ \{\mathcal{F}_A\}$ and $\mathcal{S}g^{**}\psi$ open neighbourhoods $\tilde{V}(\alpha_3)\ =\{\mathcal{F}_A\}$.

(iv) $\alpha_4=\{(e_1,\{a\}),(e2,\{d\})\ \}$. Then $\mathcal{S}g^{**}\psi$ neighbourhoods $\tilde{N}(\alpha_4)\ =\ \{\mathcal{F}_A\}$ and $\mathcal{S}g^{**}\psi$ open neighbourhoods $\tilde{V}(\alpha_4)\ =\{\mathcal{F}_A\}$.

(v) $\alpha_5=\{(e_1,\{b\}),(e2,\{d\})\}$. Then $\mathcal{S}g^{**}\psi$ neighbourhoods $\tilde{N}(\alpha_5)\ =\ \{\mathcal{F}_A\}$ and $\mathcal{S}g^{**}\psi$ open neighbourhoods $\tilde{V}(\alpha_5)\ =\{\mathcal{F}_A\}$.

(vi) $\alpha_6=\{(e_2,\{d\})\}$. Then $\mathcal{S}g^{**}\psi$ neighbourhoods $\tilde{N}(\alpha_6)\ =\ \{\mathcal{F}_A,\mathcal{F}_{A_5}\}$ and $\mathcal{S}g^{**}\psi$ open neighbourhoods $\tilde{V}(\alpha_6)\ =\{\mathcal{F}_A,\mathcal{F}_{A_5}\}$.

Remark : 4.10

A $\mathcal{Sg}^{**}\psi$ - neighbourhood generally need not be $\mathcal{Sg}^{**}\psi$ -open set. It is proved by the given example below.

By example(4.9), Let U=$\{\alpha,\beta,\gamma,\delta\}$, $E=\{J_1,J_2,J_3\}$ and $A=\{J_1,J_2\}\subseteq E$. $\mathcal{F}_A=\{(J_1,\{\alpha,\beta\}),(J_2,\{\delta\})\}$, Soft open sets $(\tilde{\tau})=\{\mathcal{F}_A,\mathcal{F}_{\varphi},\mathcal{F}_{A_5},\mathcal{F}_{A_2}\}$, Soft closed sets $(\tilde{\tau})^C=\{\mathcal{F}_A,F_{\emptyset},\mathcal{F}_{A_1},\mathcal{F}_{A_4}\}$. Then $(\mathcal{F}_A,\tilde{\tau})$ is a soft topological space. $\mathcal{Sg}^{**}\psi$ - closed sets= $\{\mathcal{F}_A,\mathcal{F}_{\varphi},\mathcal{F}_{A_1},\mathcal{F}_{A_3},\mathcal{F}_{A_4},\mathcal{F}_{A_6}\}$ and $\mathcal{Sg}^{**}\psi$ - open sets= $\{\mathcal{F}_A,\mathcal{F}_{\varphi},\mathcal{F}_{A_2},\mathcal{F}_{A_5}\}$. Consider the $\mathcal{Sg}^{**}\psi$ - neighbourhoods of F_{A_2} are $N(\alpha)=\{F_A,F_{A_3},F_{A_5}\}$. Here F_{A_3} is not a $\mathcal{Sg}^{**}\psi$ - open set.
$\alpha=F_{A_2}=\{(e_1,\{b\})\}$. Then soft g**-neighbourhoods $N(\alpha)=\{F_A,F_{A_3},F_{A_5}\}$ but it is not a soft g**-open set because F_{A_3} is not a soft g**-open set.

Theorem : 4.11

Each soft neighbourhood is also a $\mathcal{Sg}^{**}\psi$ -neighbourhood

Proof:

Let α be a soft neighbourhood of a soft set $\mathcal{F}_B\in(\mathcal{F}_A,\tilde{\tau})$, then there exists a soft open set $\mathcal{F}_C$ such that $\Rightarrow$ $\alpha\in\mathcal{F}_C\subseteq\mathcal{F}_B$. As we know that Every soft open set is $\mathcal{Sg}^{**}\psi$ -open set such that $\alpha\in\mathcal{F}_C\subseteq\mathcal{F}_B$. Hence α is $\mathcal{Sg}^{**}\psi$ - neighbourhood of $\mathcal{F}_A$.

V-REPRESENTATION OF $\mathcal{Sg}^{**}\psi$ - CLOSED SETS IN SOFT TOPOLOGY AND THEIR OPERATIONS USING BIPARTITE GRAPHS

In this section, we present a representation of soft $\mathcal{Sg}^{**}\psi$-closed sets using a bipartite graph and discuss the operations on soft $\mathcal{Sg}^{**}\psi$ - closed sets.

Definition : 5.1[4]

Every soft set can be graphically represented with the help of bipartite graph by taking partite sets A and X where A is the subset of parameter set and X is the universe.

Example 5.2[4]

X={ X_1, X_2, X_3, X_4, X_5, X_6, X_7}, E={ E_1, E_2, E_3, E_4, E_5}, A={ E_1, E_2, E_3}$\subseteq$ E

F_A ={(E_1,{ X_1, X_4, X_5}), (E_2,{ X_2, X_3, X_6}), (E_3,{ X_3, X_6})}

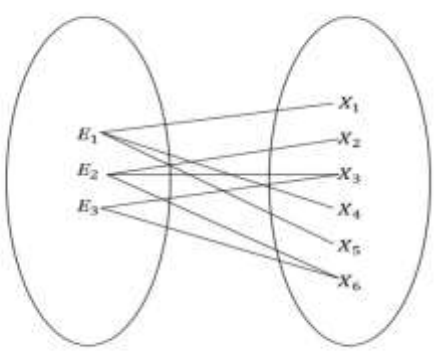

Figure 2. Soft set represented by bipartite graph"

Definition 5.3

Every $\mathcal{Sg}^{**}\psi$ -closed set can be graphically represented with the help of bipartite graph by taking partite sets A and X where A is the subset of parameter set and X is the universe.

Example 5.4

Let U={ α,β,γ}, $E=\{J_1,J_2,J_3\}$ and $A=\{J_1,J_2\}\subseteq E$, $\mathcal{F}_A=\{(J_1,\{\alpha,\beta\}),(J_2,\{\beta,\gamma\})\}$
$\mathcal{F}_{A_1}=\{(J_1,\{\alpha\})\}$, $\mathcal{F}_{A_2}=\{(J_1,\{\beta\})\}$, $\mathcal{F}_{A_3}=\{(J_1,\{\alpha,\beta\})\}$, $\mathcal{F}_{A_4}=\{(J_2,\{\beta\})\}$, $\mathcal{F}_{A_5}=\{(J_2,\{\gamma\})\}$, $\mathcal{F}_{A_6}=\{(J_2,\{\beta,\gamma\})\}$, $\mathcal{F}_{A_7}=\{(J_1,\{\alpha\}),(J_2,\{\beta\})\}$, $\mathcal{F}_{A_8}=\{(J_1,\{\alpha\}),(J_2,\{\gamma\})\}$, $\mathcal{F}_{A_9}=\{(J_1,\{\alpha\}),(J_2,\{\beta,\gamma\})\}$, $\mathcal{F}_{A_{10}}=\{(J_1,\{\beta\}),(J_2,\{\beta\})\}$, $\mathcal{F}_{A_{11}}=\{(J_1,\{\beta\}),(J_2,\{\gamma\})\}$, $\mathcal{F}_{A_{12}}=\{(J_1,\{\beta\}),(J_2,\{\beta,\gamma\})\}$, $\mathcal{F}_{A_{13}}=\{(J_1,\{\alpha,\beta\}),(J_2,\{\beta\})\}$, $\mathcal{F}_{A_{14}}=\{(J_1,\{\alpha,\beta\}),(J_2,\{\gamma\})\}$, $\mathcal{F}_{A_{15}}=\mathcal{F}_A$, $\mathcal{F}_{A_{16}}=\mathcal{F}_{\varphi}$. Soft open sets $(\tilde{\tau})=$

$\{\mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_2}, \mathcal{F}_{A_3}, \mathcal{F}_{A_{11}}, \mathcal{F}_{A_{12}}, \mathcal{F}_{A_{14}}\}$, Soft closed sets $(\tilde{\tau})^C = \{\mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_9}, \mathcal{F}_{A_6}, \mathcal{F}_{A_7}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}\}$. Then $(\mathcal{F}_A, \tilde{\tau})$ is a soft topological space , and $\mathcal{S}g^{**}\psi$ - closed sets= $\{\mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}, \mathcal{F}_{A_5}, \mathcal{F}_{A_6}, \mathcal{F}_{A_7}, \mathcal{F}_{A_8}, \mathcal{F}_{A_9}\}$.

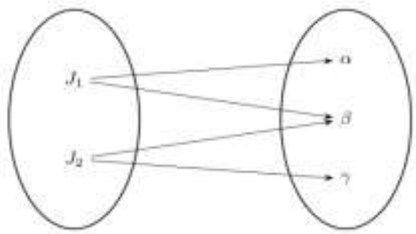

Figure 3. $\mathcal{S}g^{**}\psi$ closed-set represented by bipartite graph"

"**Property 5.5**

The union of any two $\mathcal{S}g^{**}\psi$ -closed sets admits a representation in terms of a bipartite graph.
Consider the example 5.4 Here $\mathcal{S}g^{**}\psi$ -closed sets are $\{\mathcal{F}_A, \mathcal{F}_{\varphi}, \mathcal{F}_{A_1}, \mathcal{F}_{A_4}, \mathcal{F}_{A_5}, \mathcal{F}_{A_6}, \mathcal{F}_{A_7}, \mathcal{F}_{A_8}, \mathcal{F}_{A_9}\}$. Here, we represent certain $\mathcal{S}g^{**}\psi$ unions of the $\mathcal{S}g^{**}\psi$ -closed sets.

$\mathcal{F}_{A_1} \cup \mathcal{F}_{A_4}$= $\{(J_1, \{\alpha\}), (J_2, \{\beta\})\}$ = $\mathcal{F}_{A_7}$ is also a $\mathcal{S}g^{**}\psi$ -closed set.

$\mathcal{F}_{A_7} \cup \mathcal{F}_{A_8}$ = $\{(J_1, \{\alpha\}), (J_2, \{\beta, \gamma\})\}$= $\mathcal{F}_{A_9}$ is also a $\mathcal{S}g^{**}\psi$ -closed set.

$\mathcal{F}_{A_7} \cup \mathcal{F}_{A_9}$ =$\{(J_1, \{\alpha\}), (J_2, \{\beta, \gamma\})\}$= $\mathcal{F}_{A_9}$ is also a $\mathcal{S}g^{**}\psi$ -closed set.

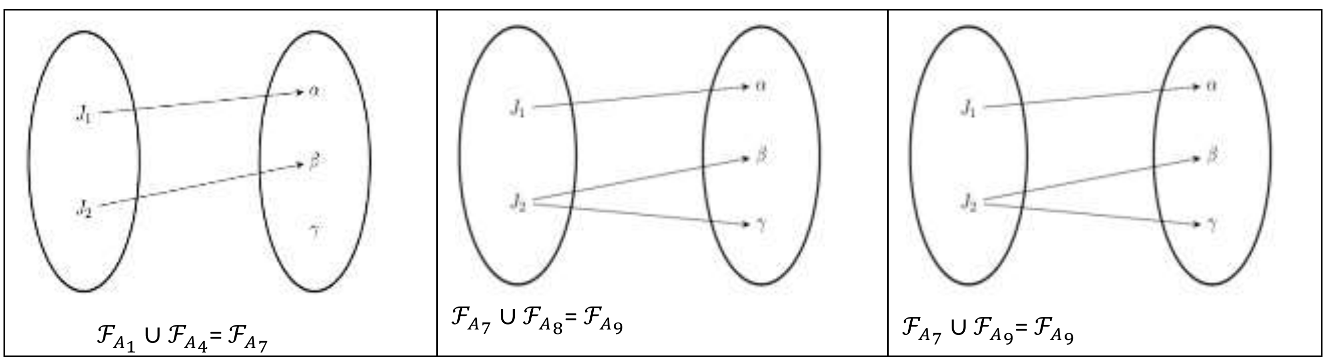

Figure 4. Union of $\mathcal{S}g^{**}\psi$ -closed set by bipartite graph

Property 5.6

Finite intersection of any two $\mathcal{S}g^{**}\psi$ -closed sets admits a representation in terms of a bipartite graph. Consider the above example 5.4 and we represent certain $\mathcal{S}g^{**}\psi$ intersections of the $\mathcal{S}g^{**}\psi$ -closed sets.

$\mathcal{F}_{A_7} \bigcap \mathcal{F}_{A_9}$ = $\{(J_1, \{\alpha\}), (J_2, \{\beta\})\}$ = $\mathcal{F}_{A_7}$ is also a $\mathcal{S}g^{**}\psi$ closed set.

$\mathcal{F}_{A_8} \bigcap \mathcal{F}_{A_9}$ = $\{(J_1, \{\alpha\}), (J_2, \{\gamma\})\}$ = $\mathcal{F}_{A_8}$is also a $\mathcal{S}g^{**}\psi$ closed set.

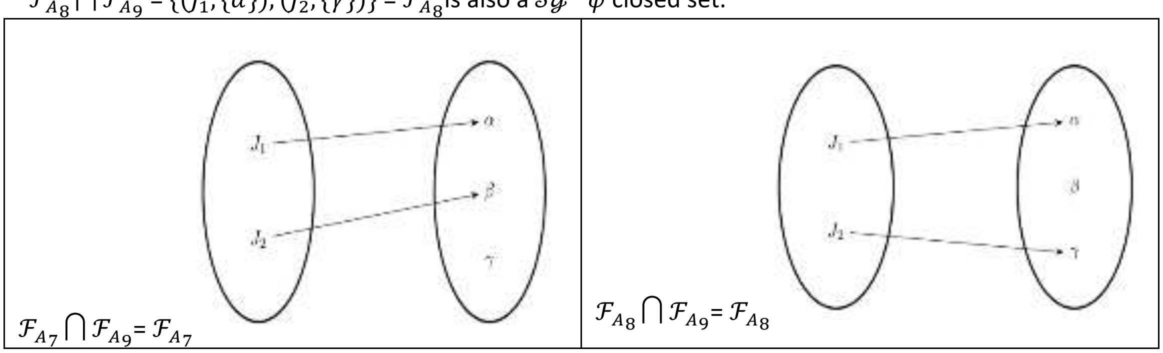

Figure 5. Intersection of $\mathcal{S}g^{**}\psi$ -closed sets by bipartite graph"

Property 5.7

The union operation involving two distinct $\mathcal{S}g^{**}\psi$ -closed sets can be illustrated as follows.

Let U=V={ $\alpha,\beta,\gamma,\delta$} , $E=\{J_1,J_2,J_3\}$ and $A=\{J_1,J_2\}\subseteq E$ and $B=\{J_2,J_3\}\subseteq E$. $\mathcal{F}_A=\{(J_1,\{\alpha,\beta\}),(J_2,\{\delta\})\}$, $\mathcal{F}_B=\{(J_2,\{\alpha\}),(J_2,\{\beta,\gamma\})\}$, $\mathcal{S}g^{**}\psi$ - closed sets of $(\mathcal{F}_A,\tilde{\tau})=\{\mathcal{F}_A,\mathcal{F}_\varphi,\mathcal{F}_{A_1},\mathcal{F}_{A_3},\mathcal{F}_{A_4},\mathcal{F}_{A_6}\}$. $\mathcal{S}g^{**}\psi$ - closed sets of $(\mathcal{F}_B,\tilde{\tau})$ ={ $\mathcal{F}_B,\mathcal{F}_\varphi,\mathcal{F}_{B_1},\mathcal{F}_{B_3},\mathcal{F}_{B_6}$}.

$\mathcal{F}_A\tilde{\bigcup}\mathcal{F}_B=\{\mathcal{F}_A,\mathcal{F}_\varphi,\mathcal{F}_{A_1},\mathcal{F}_{A_3},\mathcal{F}_{A_4},\mathcal{F}_{A_6},\ \mathcal{F}_B,\mathcal{F}_{B_1},\mathcal{F}_{B_3},\mathcal{F}_{B_6}\}$.

Where $\mathcal{F}_A=\{(J_1,\{\alpha,\beta\}),(J_2,\{\delta\})\}$, $\mathcal{F}_{A_1}=\{(J_1,\{\alpha\})\}$, $F_{A_3}=\{(J_1,\{\alpha,\beta\})\}$, $\mathcal{F}_{A_4}=\{(J_1,\{\alpha\}),(J_2,\{\delta\})\}$, $\mathcal{F}_{A_6}=\{(J_2,\{\delta\})\}$, $\mathcal{F}_B=\{(J_2,\{\alpha\}),(J_3,\{\beta,\gamma\})\}$, $\mathcal{F}_{B_1}=\{(J_2,\{\alpha\})\}$, $\mathcal{F}_{B_3}=\{(J_2,\{\alpha\}),(J_3,\{\gamma\})\}$, $\mathcal{F}_{B_6}=\{(J_3,\{\gamma\})\}$

$\Rightarrow\mathcal{F}_A\tilde{\bigcup}\mathcal{F}_B=\{(J_1,\{\alpha,\beta\}),(J_2,\{\delta\})\},\ \{(J_2,\{\alpha\}),(J_3,\{\beta,\gamma\})\},\ \{(J_3,\{\gamma\})\}\}$ can be drawn by bipartite graph

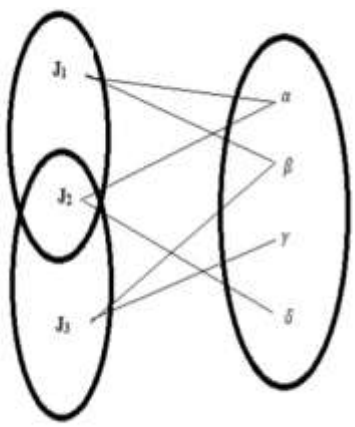

Figure 6. Union of two $\mathcal{S}g^{**}\psi$ -closed set by bipartite graph"

Property 5.8

The restricted union operation involving two distinct $\mathcal{S}g^{**}\psi$ -closed sets can be illustrated as follows from the above property 5.7

$\mathcal{F}_A\bigcup_R\mathcal{F}_B=\{(J_2,\{\alpha,\delta\})\},\{(J_3,\{\beta,\gamma\})\}$ can be represented by bipartite graph as shown below

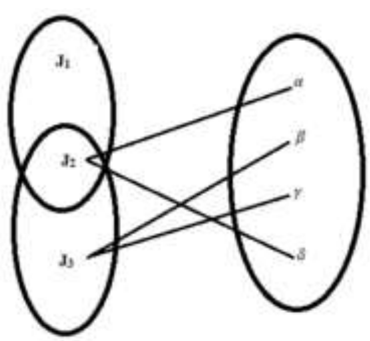

Figure 7. Restricted union of two $\mathcal{S}g^{**}\psi$ -closed set by bipartite graph

Property 5.9

The extended intersection operation involving two distinct $\mathcal{S}g^{**}\psi$ -closed sets can be illustrated as follows from the above property 5.7

$\Rightarrow$ $\mathcal{F}_{\mathrm{A}} \bigcap \varepsilon\, \mathcal{F}_{\mathrm{B}}$ $=\{\{(J_1,\{\alpha,\beta\}),(J_2,\{\delta\})\},\mathcal{F}_B=\{(J_2,\{\alpha\}),(J_3,\{\beta,\gamma\})\}\}$ can be represented by bipartite graph as shown below

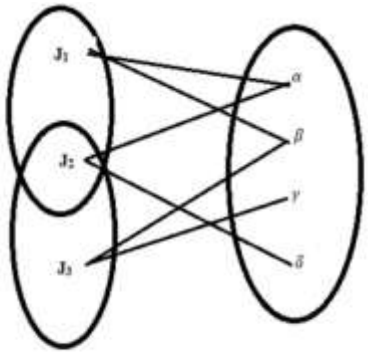

Figure 8. Extended Intersection of two $\mathcal{Sg}^{**}\psi$ -closed set by bipartite graph"

Property 5.10

AND - operation involving two distinct $\mathcal{Sg}^{**}\psi$ -closed sets can be illustrated as follows from the above property 5.7

$\mathcal{F}_A \wedge \mathcal{F}_{\mathrm{B}} \Rightarrow \mathcal{F}_{\mathrm{A}} \wedge \{\mathcal{F}_B,\mathcal{F}_{B_1},\mathcal{F}_{B_3},\mathcal{F}_{B_6}\} = \{((J_1,J_2)\{\alpha\})\}$

$\mathcal{F}_{A_1} \wedge \{\mathcal{F}_B,\mathcal{F}_{B_1},\mathcal{F}_{B_3},\mathcal{F}_{B_6}\} = \{((J_1,J_2)\{\alpha\})\}$

$\mathcal{F}_{A_3} \wedge \{\mathcal{F}_B,\mathcal{F}_{B_1},\mathcal{F}_{B_3},\mathcal{F}_{B_6}\} = \{((J_1,J_2)\{\alpha\})\}$

$\mathcal{F}_{A_4} \wedge \{\mathcal{F}_B,\mathcal{F}_{B_1},\mathcal{F}_{B_3},\mathcal{F}_{B_6}\} = \{((J_1,J_2)\{\alpha\})\}$

$\mathcal{F}_{A_6} \wedge \{\mathcal{F}_B,\mathcal{F}_{B_1},\mathcal{F}_{B_3},\mathcal{F}_{B_6}\} = \{\mathcal{F}_{\varphi}\}$

$\Rightarrow \mathcal{F}_A \wedge \mathcal{F}_{\mathrm{B}} = \{(J_1,J_2)\{\alpha\})\}$.

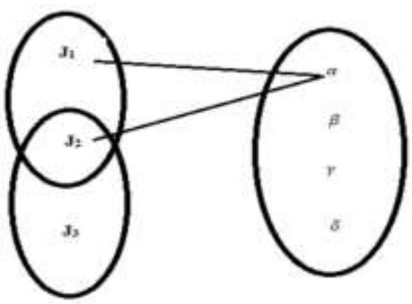

Figure 9. AND - Operation of two $\mathcal{Sg}^{**}\psi$ -closed set by bipartite graph"

Property 5.11

OR - operation involving two distinct $\mathcal{Sg}^{**}\psi$ -closed sets can be illustrated as follows from the above property 5.7

$\mathcal{F}_A \vee \mathcal{F}_{\mathrm{B}}$

$\Rightarrow \mathcal{F}_{\mathrm{A}} \vee \{\mathcal{F}_B,\mathcal{F}_{B_1},\mathcal{F}_{B_3},\mathcal{F}_{B_6}\} = \{\{((J_1,J_2)\{\alpha,\beta\}),((J_2,J_3),\{\beta,\gamma,\delta\})\}, \{((J_1,J_2)\{\alpha,\beta\}), (J_2,\{\delta\})\},$
$\{((J_1,J_2)\{\alpha,\beta\}),((J_2,J_3),\{\gamma,\delta\})\}, \{((J_1,J_3)\{\alpha,\beta,\gamma\}),(J_2,\{\delta\})\}\}$.

$\mathcal{F}_{A_1} \vee \{\mathcal{F}_B,\mathcal{F}_{B_1},\mathcal{F}_{B_3},\mathcal{F}_{B_6}\} = \{\{(J_1,J_2)\{\alpha\}),(J_3)\{\beta,\gamma\})\}, \{(J_1,J_2)\{\alpha\})\}, \{(J_1,J_2)\{\alpha\}), (J_3)\{\gamma\}\}, \{((J_1,J_3)\{\alpha,\gamma\})\}\}$.

$\mathcal{F}_{A_3} \vee \{\mathcal{F}_B,\mathcal{F}_{B_1},\mathcal{F}_{B_3},\mathcal{F}_{B_6}\} = \{\{(J_1,J_2)\{\alpha,\beta\}), ((J_3)\{\beta,\gamma\})\}, \{((J_1,J_2)\{\alpha,\beta\})\}, \{\{((J_1,J_2)\{\alpha,\beta\}),(J_3),\{\gamma\}\},$
$\{((J_1,J_3)\{\alpha,\beta,\gamma\})\}\}$.

$\mathcal{F}_{A_4} \vee \{\mathcal{F}_B,\mathcal{F}_{B_1},\mathcal{F}_{B_3},\mathcal{F}_{B_6}\} = \{\{(J_1,J_2)\{\alpha\}),((J_2,J_3),\{\beta,\gamma,\delta\})\}, \{(J_1,J_2)\{\alpha\}),((J_2),\{\delta\})\},$
$\{(J_1,J_2)\{\alpha\}),((J_2,J_3),\{\gamma,\delta\})\}, \{((J_1,J_3)\{\alpha,\gamma\}),(J_2,\{\delta\})\}\}$.

$\mathcal{F}_{A_6} \vee \{\mathcal{F}_B,\mathcal{F}_{B_1},\mathcal{F}_{B_3},\mathcal{F}_{B_6}\} = \{\{((J_1,J_2)\{\alpha,\delta\}),(J_3)\{\beta,\gamma,\delta\}\}, \{(J_2)\{\alpha,\delta\})\}, \{(J_2)\{\alpha,\delta\}),(J_3),\{\gamma\},$
$\{((J_2,J_3),\{\gamma,\delta\})\}\}$.

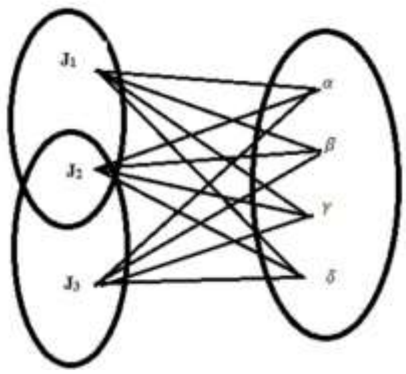

Figure 10. OR - Operation of two $\mathcal{S}g^{**}\psi$ -closed set by bipartite graph

VI-CONCLUSION

This paper introduces the concept of soft generalized**Ψ-closed sets in soft topological spaces and provides a systematic study of their structural properties. In contrast to existing approaches, the proposed class offers a meaningful generalization that extends several known types of soft closed sets, thereby enriching the framework of generalized soft topology. The relationships established between soft Ψ\PsiΨ-closed, soft g∗g^*g∗-closed, and soft generalized** Ψ\PsiΨ-closed sets demonstrate that the new class is non-trivial and exhibits distinct behavior, supported by illustrative examples and counterexamples.The investigation also includes the study of corresponding soft generalized** Ψ-open sets and their fundamental properties. Furthermore, the incorporation of bipartite graph representations provides a structured and visual interpretation of these concepts, enhancing both theoretical understanding and potential computational implementation.The results obtained in this work go beyond elementary properties and contribute to the deeper development of soft topological structures. The proposed framework opens several directions for future research, including the study of continuous mappings, compactness, and separation axioms in the context of soft generalized** Ψ\PsiΨ-closed sets. In addition, these concepts have potential applications in decision-making models, data analysis, and network-based systems where uncertainty and parameterization play a significant role.Further extensions of this work may include the exploration of these ideas in fuzzy, intuitionistic fuzzy, and nano topological settings, as well as the development of computational and graph-theoretic techniques for practical applications. Thus, the present study provides a foundation for both theoretical advancement and applied research in soft topology.

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