Research Article

A Novel Distance Based Decision Making Approach Using Fermatean Picture Fuzzy Sets For Smart Agriculture Land Suitability

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Abstract

Distance measures are fundamental tools for similarity analysis and classification in fuzzy decision-making systems. However, the existing literature does not provide Euclidean and Normalized Euclidean Distance Measures specifically developed for Fermatean Picture Fuzzy Sets (FPFS), limiting their application in distance-based intelligent decision-making. To address this limitation, this paper proposes new Euclidean and Normalized Euclidean Distance Measures for FPFS and establishes their essential mathematical properties. The effectiveness of the proposed measures is demonstrated through a K-Nearest Neighbour (KNN)-based agricultural land suitability assessment under the Fermatean picture fuzzy environment. The obtained results show that both measures accurately evaluate the similarity among alternatives and produce consistent classification outcomes. Moreover, the normalized measure expresses distance values within a common interval, enabling meaningful comparison while preserving the relative ranking of alternatives. The proposed framework provides a mathematically sound and computationally efficient approach for similarity analysis and intelligent decision-making under Fermatean Picture Fuzzy information.

Keywords: Fermatean Picture Fuzzy Sets, Euclidean Distance Measure, Normalized Euclidean Distance Measure, K-Nearest Neighbour, Agricultural Land Suitability, Similarity Analysis.

1. Introduction

Decision-making in engineering, healthcare, environmental management, agriculture, finance, and artificial intelligence often involves uncertain, vague, and incomplete information. Since conventional mathematical approaches rely on precise numerical data, they are frequently inadequate for modelling such uncertainty. The introduction of fuzzy set theory by Zadeh [23,24] provided a powerful framework for representing imprecise information through membership functions and has become the foundation for numerous developments in fuzzy mathematics and intelligent decision-making.

To improve the representation of uncertainty, several extensions of fuzzy sets have been proposed over the years. Atanassov introduced intuitionistic fuzzy sets by incorporating both membership and non-membership degrees, thereby enhancing the capability of fuzzy models to represent incomplete information [3]. Subsequently, Yager proposed Pythagorean fuzzy sets, where the squared sum of the membership and non-membership degrees is restricted to one, allowing a larger domain for expressing uncertainty [21,22]. Senapati and Yager further generalized this concept through Fermatean fuzzy sets by replacing the squared constraint with a cubic one, thereby enlarging the admissible decision space and improving modelling flexibility in uncertain environments [1,2,5,13,19].

Although Fermatean fuzzy sets provide greater flexibility in representing uncertainty, many practical decision-making problems also involve neutral or indeterminate opinions. To accommodate this requirement, Cuong introduced picture fuzzy sets by considering positive, neutral, and negative membership degrees simultaneously [10]. Owing to this three-dimensional representation, picture fuzzy sets have been successfully employed in applications such as pattern recognition, medical diagnosis, image analysis, and multi-criteria decision-making [11,12].

Fermatean Picture Fuzzy Sets (FPFS) combine the expressive capability of Fermatean fuzzy sets with the descriptive power of picture fuzzy sets, providing a richer framework for representing uncertain information. Recent studies have expanded the theoretical development of FPFS through investigations on topological structures, continuity, connectedness, and compactness, demonstrating their effectiveness in handling complex mathematical and decision-making problems [6–8].

Among the numerous mathematical tools used in fuzzy decision-making, distance measures occupy a central position because they quantify the degree of dissimilarity between uncertain objects. They form the basis for similarity analysis, clustering, pattern recognition, medical diagnosis, and classification [18,25]. Various distance measures, including Hamming, Hausdorff, weighted, and Euclidean distances, have been developed for intuitionistic, neutrosophic, and picture fuzzy environments [4,14,16,20,22]. Due to its geometric interpretation, computational simplicity, and effectiveness, the Euclidean distance remains one of the most widely adopted measures in similarity-based decision-making.

For meaningful comparison among alternatives, normalization of distance values is equally important. A normalized distance measure transforms distance values into a common interval, typically [0,1], thereby facilitating consistent comparison among alternatives irrespective of scale. Despite the rapid development of Fermatean Picture Fuzzy Sets, the literature lacks a dedicated study proposing both a Euclidean Distance Measure and a corresponding Normalized Euclidean Distance Measure for FPFS. This limitation restricts the use of FPFS in similarity analysis, pattern recognition, and distance-based machine learning applications.

Agricultural land suitability assessment is a representative multi-criteria decision-making problem in which several interrelated factors, including soil characteristics, climatic conditions, water availability, terrain, and nutrient content, must be evaluated simultaneously. Since these factors are often associated with uncertainty and subjective expert judgment, fuzzy approaches have been widely adopted for agricultural decision support [15,17]. Furthermore, the K-Nearest Neighbour (KNN) algorithm, introduced by Cover and Hart [9], is a well-established distance-based classification technique that assigns class labels according to the similarity between observations. Therefore, KNN provides an appropriate framework for evaluating the effectiveness of newly developed distance measures under the Fermatean Picture Fuzzy environment.

Motivated by these observations, this paper proposes a Euclidean Distance Measure and a Normalized Euclidean Distance Measure for Fermatean Picture Fuzzy Sets. The proposed measures satisfy the fundamental properties required of valid distance measures, namely non-negativity, identity, symmetry, and boundedness. Their practical applicability is demonstrated through a KNN-based agricultural land suitability assessment, where alternatives represented by Fermatean Picture Fuzzy information are classified according to their similarity. The obtained results demonstrate that the proposed measures provide an effective, mathematically consistent, and computationally efficient framework for similarity analysis and intelligent decision-making under uncertain environments.

The major contributions of this paper are summarized as follows:

- A novel Euclidean Distance Measure is proposed for Fermatean Picture Fuzzy Sets.
- A corresponding Normalized Euclidean Distance Measure is developed to enable standardized similarity evaluation.
- The fundamental mathematical properties of the proposed measures are established.
- A KNN-based classification framework is developed using the proposed distance measures.
- The effectiveness of the proposed approach is validated through a smart agricultural land suitability assessment under the Fermatean Picture Fuzzy environment.

The remainder of this paper is organized as follows. Section 2 presents the preliminary concepts related to Fermatean Picture Fuzzy Sets. Section 3 introduces the proposed Euclidean and Normalized Euclidean Distance Measures. Section 4 establishes their mathematical properties and presents the KNN-based classification framework. Section 5 demonstrates the applicability of the proposed approach through an agricultural land suitability case study. Finally, Section 6 concludes the paper and outlines possible directions for future research.

2. Preliminaries

This section starts by revisiting some key definitions.

Definition 2.1. [23]

Let X be a universe of discourse, then a fuzzy set A can be defined as an entity characterized by the following formulation: $A = \{\langle x, \mu_A(x)\rangle | x \in X\}$, where $\mu_A: X \to [0,1]$ and $\mu_A(x)$ is called the membership degree of x in X.

Definition 2.2. [3]

Let U be a universe. An intuitionistic fuzzy set A on of the universe U is given as:
$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$, where $\mu_A: U \to [0,1]$ and $\nu_A: X \to [0,1]$ with $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all $x \in U$, with $\mu_A(x)$ and $\nu_A(x)$ are the degree of membership and degree of nonmembership of the element x respectively.

Definition 2.3. [21]

Suppose that X is any non-empty set and I is the unit interval [0,1]. A Pythagorean fuzzy set A is an object defined by the expression $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$, where $\mu_A: U \to [0,1]$ and $\nu_A: X \to [0,1]$ represents the degree of membership and degree of non-membership of each element $x \in X$ to the set A, such that $(\mu_A(x))^2 + (\nu_A(x))^2 \le 1$ for each $x \in X$.

Definition 2.4. [19]

Suppose that X is a universal set. A Fermatean fuzzy set (FFS) F defined on X is said to be a structure of the type $F = \{(x, \alpha_F(x), \beta_F(x))/x \in X\}$, where $\alpha_F(x) : X \to [0,1]$ and $\beta_F(x) : X \to [0,1]$, such that $0 \le (\alpha_F(x))^3 + (\beta_F(x))^3 \le 1$, for all $x \in X$. The values $\alpha_F(x)$ and $\beta_F(x)$ represent, respectively, the degree of membership and the degree of non-membership of the element x in the set F. For any FFS F and

$x \in X, \pi_F(x) = \sqrt[3]{1-(\alpha_F(x))^3-(\beta_F(x))^3}$ represents the degree of indeterminacy of x to F.

Definition 2.5. [10]

Let A be the Picture Fuzzy set (PFS) defined on a universe U as,

$A = \{(u, \mu_A(u), \eta_A(u), \gamma_A(u))/u \in U\}$, where $\mu_A(u), \eta_A(u), \gamma_A(u)$ denoted as the degree of positive membership, the degree of neutral membership, the degree of negative membership of u in A respectively with the constraint $0 \leq \mu_A(u) + \eta_A(u) + \gamma_A(u) \leq 1, \forall u \in U$.

Therefore, $\forall u \in U$, we have $\pi_A(u) = 1 - (\mu_A(u) + \eta_A(u) + \gamma_A(u))$ is called the degree of refusal membership of u in A.

Definition 2.6. [7]

A Fermatean Picture Fuzzy Set G on a universal set W is defined as

$G = \{(j, \alpha_G(j), \beta_G(j), \gamma_G(j)/j \in W)\}$, where $\alpha_G(j), \beta_G(j), \gamma_G(j)$ represent the degree of positive membership, the degree of neutral membership and the degree of negative membership of j in G and the following conditions are met.

$0 \leq \alpha_G(j), \beta_G(j), \gamma_G(j) \leq 1$, and $0 \leq {\alpha_G}^3(j) + {\beta_G}^3(j) + {\gamma_G}^3(j) \leq 1, \forall j \in W$.

Then $\forall j \in W, \pi_G(j) = 1 - ({\alpha_G}^3(j) + {\beta_G}^3(j) + {\gamma_G}^3(j))$ denotes the degree of refusal membership of j in G.

Definition 2.7. [7]

Let W be a non-empty set and the FPFS G and H be in the form

$G = \{(j, \alpha_G(j), \beta_G(j), \gamma_G(j)/j \in W)\}$ and $H = \{(j, \alpha_H(j), \beta_H(j), \gamma_H(j)/j \in W)\}$. Then

1. $G \subseteq H$ iff $\alpha_G(j) \leq \alpha_H(j), \beta_G(j) \leq \beta_H(j)$ and $\gamma_G(j) \geq \gamma_H(j)$
2. $G = H$ iff $\alpha_G(j) = \alpha_H(j), \beta_G(j) = \beta_H(j)$ and $\gamma_G(j) = \gamma_H(j)\ \forall j \in W$
3. $G \cap H = \{(j, \alpha_{G\cap H}(j), \beta_{G\cap H}(j), \gamma_{G\cap H}(j)) | j \in W\}$ where
   i. $\alpha_{G\cap H}(j) = min\{\alpha_G(j), \alpha_H(j)\}$
   ii. $\beta_{G\cap H}(j) = min\{\beta_G(j), \beta_H(j)\}$
   iii. $\gamma_{G\cap H}(j) = max\{\gamma_G(j), \gamma_H(j)\}$
4. $G \cup H = \{(j, \alpha_{G\cup H}(j), \beta_{G\cup H}(x), \gamma_{G\cup H}(j)) | j \in W\}$ where
   i. $\alpha_{G\cup H}(j) = max\{\alpha_G(j), \alpha_H(j)\}$
   ii. $\beta_{G\cup H}(j) = min\{\beta_G(j), \beta_H(j)\}$
   iii. $\gamma_{G\cup H}(j) = min\{\gamma_G(j), \gamma_H(j)\}$

Definition 2.8. [7]

Let $G = \{(j, \alpha_G(j), \beta_G(j), \gamma_G(j)/j \in W)\}$ is a Fermatean Picture Fuzzy set on W, then the complement of the set G ($\overline{G}$) may be defined as

$$\overline{G} = \left\{\left(j, \gamma_G(j), \sqrt[3]{1 - (\alpha_G^3(j) + \beta_G^3(j) + \gamma_G^3(j))}, \alpha_G(j)\right) / j \in W\right\}$$

Definition 2.9. Euclidean distance (dE_{PS}) [12]

Let $P = \{(x, \mu_P(x), \eta_P(x), \nu_P(x)/x \in X)\}$ and $Q = \{(x, \mu_Q(x), \eta_Q(x), \nu_Q(x))/x \in X\}$ be two picture fuzzy sets defined on the finite universe $X = \{x_1, x_2, \ldots, x_n\}$. Here, $\mu(x), \eta(x)$, and $\nu(x)$ denote the positive, neutral, and negative membership degrees, respectively, satisfying $0 \leq \mu(x) + \eta(x) + \nu(x) \leq 1$.

The Euclidean distance between P and Q is defined as

$$dE_{PS}(P,Q) = \sqrt{\sum_{i=1}^{n} [\left(\mu_P(x_i) - \mu_Q(x_i)\right)^2 + \left(\eta_P(x_i) - \eta_Q(x_i)\right)^2 + \left(\nu_P(x_i) - \nu_Q(x_i)\right)^2]}.$$

Definition 2.10. Normalized Euclidean distance (dNE_{PS}) [11]

Let $P = \{(x, \mu_P(x), \eta_P(x), \nu_P(x)/x \in X)\}$ and $Q = \{(x, \mu_Q(x), \eta_Q(x), \nu_Q(x))/x \in X\}$ be two picture fuzzy sets defined on the finite universe $X = \{x_1, x_2, \ldots, x_n\}$. Here, $\mu(x), \eta(x)$, and $\nu(x)$ denote the positive, neutral, and negative membership degrees, respectively, satisfying $0 \leq \mu(x) + \eta(x) + \nu(x) \leq 1$.

The Normalized Euclidean distance between P and Q is defined as

$$dNE_{PS}(P,Q) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n} [\left(\mu_P(x_i) - \mu_Q(x_i)\right)^2 + \left(\eta_P(x_i) - \eta_Q(x_i)\right)^2 + \left(\nu_P(x_i) - \nu_Q(x_i)\right)^2]}.$$

The above definitions establish the mathematical framework required for developing the proposed distance measures.

3. Distance Measures for Fermatean Picture Fuzzy sets

Definition 3.1 Euclidean distance Measure for Fermatean Picture Fuzzy sets (dE_{FPFS})
Let W be a non-empty set and the FPFS G and H be in the form
$G = \{(j, \alpha_G(j), \beta_G(j), \gamma_G(j)/j \in W)\}$ and $H = \{(j, \alpha_H(j), \beta_H(j), \gamma_H(j)/j \in W)\}$. Then Euclidean Distance Measure between G and H is defined as

$$dE_{FPFS}(G,H) = \sqrt{\sum_{j=1}^{n} [(\alpha_G^3(j) - \alpha_H^3(j))^2 + (\beta_G^3(j) - \beta_H^3(j))^2 + (\gamma_G^3(j) - \gamma_H^3(j))^2]}.$$

Definition 3.2 Normalized Euclidean distance Measure for Fermatean Picture Fuzzy sets (dNE_{FPFS})
Let W be a non-empty set and the FPFS G and H be in the form
$G = \{(j, \alpha_G(j), \beta_G(j), \gamma_G(j)/j \in W)\}$ and $H = \{(j, \alpha_H(j), \beta_H(j), \gamma_H(j)/j \in W)\}$. Then Euclidean Distance Measure between G and H is defined as

$$dNE_{FPFS}(G,H) = \sqrt{\frac{1}{3n}\sum_{j=1}^{n} \left[(\alpha_G^3(j) - \alpha_H^3(j))^2 + (\beta_G^3(j) - \beta_H^3(j))^2 + (\gamma_G^3(j) - \gamma_H^3(j))^2\right]}$$

Property 3.3.
For any two Fermatean Picture Fuzzy sets G and H, the following properties are satisfied.
(i) $dE_{FPFS}(G,H) \geq 0$
(ii) $dE_{FPFS}(G,H) = 0$ if and only if $G = H$.
(iii) $dE_{FPFS}(G,H) = dE_{FPFS}(H,G)$
(iv) $0 \leq dE_{FPFS}(G,H) \leq 1$
Proof:
Let W be a non-empty set and the FPFS G and H be in the form
$G = \{(j, \alpha_G(j), \beta_G(j), \gamma_G(j)/j \in W)\}$ and $H = \{(j, \alpha_H(j), \beta_H(j), \gamma_H(j)/j \in W)\}$.
(i) Since $(\alpha_G^3(j) - \alpha_H^3(j))^2 \geq 0, (\beta_G^3(j) - \beta_H^3(j))^2 \geq 0$ and $(\gamma_G^3(j) - \gamma_H^3(j))^2 \geq 0$
For every $j \in W$, their sum also non negative. Therefore $dE_{FPFS}(G,H) \geq 0$.
(ii) Suppose $dE_{FPFS}(G,H) = 0$, then
$\sum_{j=1}^{n} [(\alpha_G^3(j) - \alpha_H^3(j))^2 + (\beta_G^3(j) - \beta_H^3(j))^2 + (\gamma_G^3(j) - \gamma_H^3(j))^2] = 0$.
Hence $\alpha_G^3(j) = \alpha_H^3(j)$, $\beta_G^3(j) = \beta_H^3(j)$ and $\gamma_G^3(j) = \gamma_H^3(j)$ for every $j \in W$.
Since the cube function is one-to-one on [0,1], then $\alpha_G(j) = \alpha_H(j)$, $\beta_G(j) = \beta_H(j)$ and $\gamma_G(j) = \gamma_H(j)$.
Therefore $G = H$.
Conversely, if $G = H$, every squared difference is zero. Hence $dE_{FPFS}(G,H) = 0$.
(iii) Observe $(\alpha_G^3(j) - \alpha_H^3(j))^2 = (\alpha_H^3(j) - \alpha_G^3(j))^2$
Likewise, $(\beta_G^3(j) - \beta_H^3(j))^2 = (\beta_H^3(j) - \beta_G^3(j))^2$
and $(\gamma_G^3(j) - \gamma_H^3(j))^2 = (\gamma_H^3(j) - \gamma_G^3(j))^2$.
Thus, $dE_{FPFS}(G,H) = dE_{FPFS}(H,G)$.
(iv) From (i), $dE_{FPFS}(G,H) \geq 0$,
Since $0 \leq \alpha_G(j), \beta_G(j), \gamma_G(j) \leq 1$, their cubes also belong to the interval [0,1].
Hence, $(\alpha_G^3(j) - \alpha_H^3(j))^2 \leq 1$, $(\beta_G^3(j) - \beta_H^3(j))^2 \leq 1$ and $(\gamma_G^3(j) - \gamma_H^3(j))^2 \leq 1$.
Therefore, $[(\alpha_G^3(j) - \alpha_H^3(j))^2 + (\beta_G^3(j) - \beta_H^3(j))^2 + (\gamma_G^3(j) - \gamma_H^3(j))^2] \leq 3$.
Summing over all n elements, $\sum_{j=1}^{n} \left[(\alpha_G^3(j) - \alpha_H^3(j))^2 + (\beta_G^3(j) - \beta_H^3(j))^2 + (\gamma_G^3(j) - \gamma_H^3(j))^2\right] \leq 3n$.
Taking the square root of both sides,
$\sqrt{\sum_{j=1}^{n} [(\alpha_G^3(j) - \alpha_H^3(j))^2 + (\beta_G^3(j) - \beta_H^3(j))^2 + (\gamma_G^3(j) - \gamma_H^3(j))^2]} \leq \sqrt{3n}$.
Thus, $0 \leq dE_{FPFS}(G,H) \leq \sqrt{3n}$. Then it is provable that, $0 \leq dE_{FPFS}(G,H) \leq 1$.
Property 3.4.
Let G and H be two Fermatean Picture Fuzzy Sets defined on the universe W. Then, the proposed Normalized Euclidean distance measure possesses the following fundamental properties.
(i) $dNE_{FPFS}(G,H) \geq 0$
(ii) $dNE_{FPFS}(G,H) = 0$ if and only if $G = H$.
(iii) $dNE_{FPFS}(G,H) = dNE_{FPFS}(H,G)$
(iv) $0 \leq dNE_{FPFS}(G,H) \leq 1$

Proof: Obvious

Remark 3.5.

The refusal membership degree is not explicitly included in the proposed distance measure since it is uniquely determined by the positive, neutral, and negative membership degrees through

$$\pi_G(j) = 1 - ({\alpha_G}^3(j) + {\beta_G}^3(j) + {\gamma_G}^3(j))$$

Therefore, any variation in the refusal degree is implicitly reflected through the corresponding variations in the three independent membership components. Consequently, incorporating the refusal degree separately would introduce redundant information without providing additional discriminatory power.

4. FPFS-KNN Algorithm for Smart Agricultural Land Suitability Assessment

4.1 Problem Description

The objective is to classify agricultural lands into different suitability categories based on multiple criteria under the Fermatean Picture Fuzzy environment. Since agricultural data often contain uncertainty, hesitation, neutrality, and conflicting opinions from experts, the proposed FPFS-KNN algorithm employs the newly developed Euclidean distance measure to identify the nearest neighboring lands and predict the suitability class of an unknown land.

4.2 Input Data

Define $A = A_1, A_2, A_3 \dots \dots . A_m$ as agricultural lands.

Define $C = C_1, C_2, C_3 \dots \dots . C_n$ as evaluation criteria.

4.3 FPFS Representation

Convert every criterion into FPFS.

4.4 Proposed FPFS-KNN Algorithm

Step 1: Construct FPFS training data.

Step 2: Represent unknown land using FPFS.

Step 3: Compute the proposed Euclidean distance and Normalized Euclidean distance

Step 4: Sort distances.

Step 5: Select K nearest neighbors.

Step 6: Majority Voting.

Step 7: Predict land suitability.

4.5 Mathematical Model

1. Write Training set $T = \{(A_i, L_i)\}$, where

- A_i = i th agricultural land (feature vector)
- L_i = class label (Highly Suitable, Moderately Suitable, Unsuitable)

2. Then for an unknown land A, Compute the Distance $dE_{FPFS}(A, A_i)$ and $dNE_{FPFS}(A, A_i)$

3. Choose Nearest neighbors $N_K(A)$

4. Prediction $\hat{L} = argmax \sum I(L_i)$

i.e the class having the maximum number of votes among the K nearest neighbours.

5. Case Study: Smart Agricultural Land Suitability Assessment

5.1 Description of the Problem

The selection of suitable agricultural land is a crucial task in precision agriculture. Land suitability depends on multiple environmental and soil-related factors, which often involve uncertainty due to varying expert opinions and incomplete information. To address this challenge, the proposed Euclidean Distance Measure and Normalized Euclidean Distance Measure for Fermatean Picture Fuzzy Sets are employed within a K-Nearest Neighbour (KNN) framework to classify agricultural lands into different suitability categories.

The dataset used in this study is adapted from the Food and Agriculture Organization (FAO) Land Evaluation Framework and published agricultural suitability studies. The original crisp measurements are transformed into Fermatean Picture Fuzzy values based on expert-defined linguistic assessments.

5.2 Criteria Selection

Six important criteria are considered.

Criteria	Symbol
Soil pH	C_1
Rainfall	C_2

Temperature	C_3
Soil Organic Carbon	C_4
Water Availability	C_5
Drainage	C_6

5.3 Secondary Dataset (Adapted)

Land	Soil pH	Rainfall (mm)	Temp (°C)	Organic Carbon (%)	Water Index	Drainage
A_1	6.6	840	27	0.85	8	Good
A_2	7.1	900	26	0.91	9	Excellent
A_3	5.7	620	34	0.43	4	Poor
A_4	6.8	790	29	0.72	7	Moderate
A_5	6.3	720	31	0.65	6	Moderate

5.4 Linguistic Scale

Crisp Condition	Linguistic Term
Excellent	Very High
Good	High
Average	Medium
Poor	Low
Very Poor	Very Low

5.5 FPFS Decision Matrix

Using expert opinions, the linguistic values are converted into FPFS numbers.

Land	Soil pH	Rainfall	Temperature	Organic Carbon	Water	Drainage
A_1	(0.88,0.18,0.15)	(0.90,0.15,0.12)	(0.86,0.18,0.16)	(0.89,0.15,0.14)	(0.87,0.17,0.15)	(0.90,0.12,0.11)
A_2	(0.93,0.10,0.08)	(0.94,0.09,0.07)	(0.91,0.11,0.10)	(0.95,0.08,0.06)	(0.93,0.10,0.08)	(0.96,0.05,0.05)
A_3	(0.56,0.32,0.40)	(0.52,0.35,0.44)	(0.58,0.28,0.39)	(0.50,0.34,0.45)	(0.54,0.30,0.42)	(0.48,0.35,0.47)
A_4	(0.80,0.20,0.18)	(0.82,0.18,0.16)	(0.81,0.18,0.18)	(0.79,0.20,0.19)	(0.80,0.18,0.18)	(0.82,0.17,0.16)
A_5	(0.72,0.25,0.23)	(0.74,0.23,0.22)	(0.71,0.24,0.24)	(0.73,0.22,0.21)	(0.70,0.25,0.24)	(0.72,0.23,0.22)

5.6 Training Dataset

Land	Description	Suitability Class
A_1	Land with favourable soil properties, good rainfall, high organic carbon content, and good drainage	Highly Suitable
A_2	Land with excellent soil fertility, optimum climatic conditions, highest water availability, and excellent drainage	Highly Suitable
A_3	Land with poor soil fertility, inadequate rainfall, high temperature, and poor drainage	Unsuitable

A_4	Land with moderately favourable soil and water conditions and moderate drainage	Moderately Suitable
A_5	Land with acceptable agricultural conditions but moderate limitations in soil and water availability	Moderately Suitable

5.7 Test Land

Suppose a new agricultural land A_6 has the following FPFS values.

Criterion	**FPFS Value**
Soil pH	(0.84,0.18,0.16)
Rainfall	(0.86,0.16,0.14)
Temperature	(0.83,0.18,0.17)
Organic Carbon	(0.85,0.16,0.15)
Water Availability	(0.84,0.17,0.16)
Drainage	(0.86,0.15,0.14)

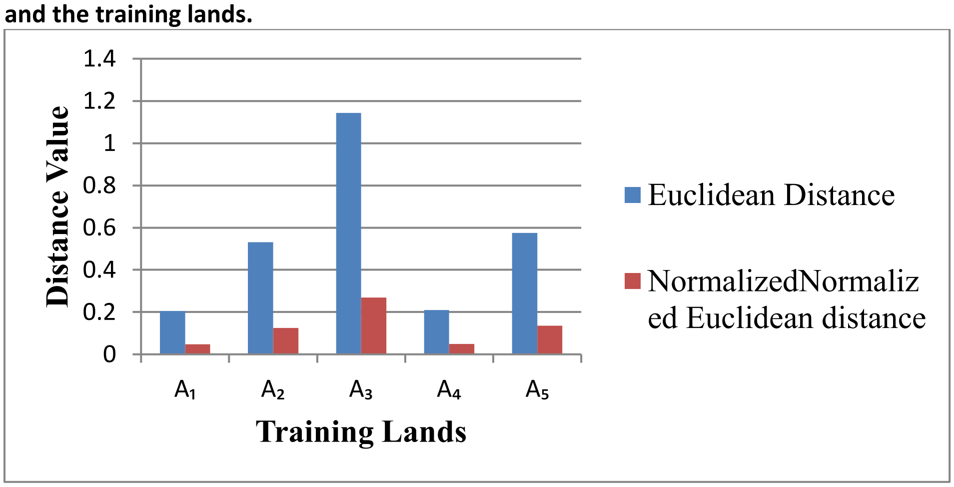
5.8 Distance Computation

Euclidean distance and Normalized Euclidean distance between A_6 and each training land:

	Euclidean Distance (dE_{FPFS}(A_6,A_i))	**Normalized Euclidean distance (dNE_{FPFS}(A_6,A_i))**
A_1	0.204642282	0.048234648
A_2	0.530707192	0.125088885
A_3	1.143696804	0.269571922
A_4	0.209066583	0.049277466
A_5	0.574577844	0.135429297

The above table shows that both the proposed Euclidean Distance Measure and the proposed Normalized Euclidean Distance Measure produce the same ranking of the training lands. The test land A_6 is most similar to A_1 and least similar to A_3. The normalization process only rescales the distance values and does not affect the similarity ordering. Hence, both measures identify the same nearest neighbours for KNN classification.

Figure 1. Comparison of the Euclidean Distance and Normalized Euclidean Distance between the test land and the training lands.



5.9 KNN Classification (K=3)
The three nearest neighbours are:

Rank	Land	Class
1	A_1	Highly Suitable
2	A_4	Moderately Suitable
3	A_2	Highly Suitable

Using majority voting:
- Highly Suitable = 2 votes
- Moderately Suitable = 1 vote

Since two of the three nearest neighbours belong to the Highly Suitable class, the test sample is classified as Highly Suitable according to the majority voting principle.
Therefore, $\boldsymbol{Predicted\ class}(\mathbf{A_6}) = \boldsymbol{Highly\ Suitable}$

5.10 Discussion
The proposed Euclidean Distance Measure and Normalized Euclidean Distance Measure were successfully applied to an agricultural land suitability assessment under the Fermatean Picture Fuzzy environment. The computed distance values effectively measured the similarity between the test land and the training lands. Both distance measures produced the same ranking of alternatives and identified the same nearest neighbours, resulting in a consistent KNN classification of the test land as Highly Suitable. The case study shows how the suggested distance measurements give a dependable and efficient way of dealing with uncertainty in agriculture and can also be used in other multicriteria decision making problems.

6. Conclusion and Future Work

This paper presented two novel distance measures, namely the Euclidean Distance Measure (EDM) and the Normalized Euclidean Distance Measure (NEDM), for Fermatean Picture Fuzzy Sets (FPFS). The proposed measures were mathematically formulated, and their fundamental properties, including non-negativity, identity, symmetry, and boundedness, were established. By extending the concept of Euclidean distance to the FPFS environment, the proposed measures provide an effective framework for evaluating similarity among alternatives represented by positive, neutral, and negative membership information.
To demonstrate their practical applicability, the proposed measures were incorporated into a K-Nearest Neighbour (KNN) classification framework for smart agricultural land suitability assessment. The experimental results showed that both distance measures produced consistent similarity rankings and accurate classification outcomes. In particular, the normalized Euclidean distance expressed similarity values within a standardized interval while preserving the relative ordering of alternatives, thereby facilitating meaningful comparison and interpretation. These results confirm that the proposed framework is mathematically reliable, computationally efficient, and suitable for intelligent decision-making under uncertain environments.
Future research may focus on extending the proposed distance measures to other Fermatean Picture Fuzzy models and developing corresponding similarity, entropy, and divergence measures. In addition, integrating the proposed framework with advanced machine learning and deep learning techniques may further enhance its applicability to complex decision-making problems in healthcare, pattern recognition, image analysis, financial risk assessment, recommendation systems, and other intelligent decision-support applications.

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