

Analysis of An M/G/1 Retrial G-Queue With Various Types of Customers Under Working Vacation

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ABSTRACT:

Consider a single server retrial queueing system with preemptive priority service and single working vacation in which two types of customers arrive in a Poisson process with arrival rates. (i) ordinary customers (ii) for high level Priority customers, we assumed that regular service times follows an exponential distribution with parameters μ_1 and μ_2 . The retrial is introduced for low priority customers only. During working vacation the server serve's the arriving customers with less service rates μ_3 and μ_4 .

An M/G/1 retrial G-queue with preemptive priority for primary customers models a single –server system where high- priority primary customers immediately interrupt regular service, while low-priority (or) blocked customers enter a retrial orbit and negative customers (“G-Customers”).

The negative customer is arriving during the service time of any positive customer, will remove the positive customer from the service. If the interrupted customer is an ordinary customer, he will leave the system. As soon as the orbit becomes empty at the time of service completion, the server goes for a working vacation. The server works at a lower speed during a working vacation period.

Using the supplementary variable technique, the steady –state probability generating a function of the system and its orbit are found some numerical examples are presented.

KEYWORDS: Retrial queue, priority queue, negative customers, working vacations, supplementary variable technique.

Introduction

ARRIVAL PROCESS:

Assume that three types of customers arrive from outside the system using independent Poisson processes: regular, priority and disaster. Preemptive priority customers have dominated the regular busy server during the service time of regular customers.

THE REGULAR SERVICE PROCESS:

The regular busy server serves an ordinary customer; if priority customers arrive, then the regular service will be disrupted. A priority customer takes precedence over an ordinary customer and the server begins its service immediately. The typical customer must wait in the service area for the remainder of the service to be completed.

PRIORITY SERVICE PROCESS:

When priority customers arrive, they find the server accessible and immediately begin its service. Assume that if the server provides service to another priority customer, the newly arriving priority customer will exit the system without receiving service.

THE REMOVAL RULE:

Negative (failure) customers arrive only during regular service hours. This kind of arrival will push regular customers (priority or ordinary) out of the service area and cause the server to fail. When a server fails, it is immediately sent to be repaired. When the repair process is finished, the server will function as if it were brand new.

THE WORKING VACATION PROCESS:

When no customers are in orbit, the server initiates a working vacation. During the working vacation period, the server serves at a slower service rate. If any customer is available in orbit at the end of the Working vacation, the server will switch from the lesser service to the normal busy period. Otherwise, the server either waits with probability p to serve a new customer (a single working vacation) or takes another working vacation with

probability $q_1 - p$ (a multiple working vacations). If customers are in orbit at the end of the vacation, the server returns to its normal working state.

PRACTICAL EXAMPLE OF THE MODEL:

We consider a telecommunication system one of the best examples of our model. Telecommunication, banks, call centers, and mobile network customers play a significant role in this COVID-19 pandemic. All the network operators work only on emergency cases with a limited number of workers. People can contact the center by communicating with a person at the help desk or customer service consultant (CSC) through the telephone (regular server). Customers can contact the center via fax, e-mail, or live chat sessions in addition to calling (priority customers). The CSC or agent picks them both up. If the agent is engaged at the time of the call, the arriving call will lose service. However, if the agent is concerned

About emails or faxes, the call takes precedence over an e-mail service. The preempted service will remain in place to complete its mission. If customers are dissatisfied with the service, they will be re-served as soon as possible (re-service). The working server will be impacted by a weak signal (negative customers), causing the system to fail during the service period and need to be repaired immediately.

If CSC gets no voice calls, faxes, or emails, it will execute some maintenance jobs to enhance the computer performance (working vacation). If an arriving message is discovered while the CSC is engaged in a voice call, the messages are temporarily stored in a retrieval buffer (orbit) and served after some time (retrial time). This queueing discipline is a good approximation of this type of a telecommunication processing system.

THE STEADY STATE EQUATIONS:

In steady state, we assume that $R(0) = 0$, $R(\infty) = 1$, $S_p(0) = 0$, $S_p(\infty) = 1$, $S_b(0) = 0$, $S_b(\infty) = 1$, $S_v(0) = 0$, $S_v(\infty) = 1$, are continuous at $x = 0$. So that the function $a(x)$, $\mu_p(x)$, $\mu_b(x)$ and $\mu_v(x)$ are the conditional completion rates (hazard rate) for retrial, service of a priority customer and ordinary customer, lower rates service respectively.

$$a(x)dx = \frac{dR(x)}{1-R(x)}, \mu_p(x)dx = \frac{dS_p(x)}{1-S_p(x)}, \mu_b(x)dx = \frac{dS_b(x)}{1-S_b(x)}, \mu_v(x)dx = \frac{dS_v(x)}{1-S_v(x)}.$$

Let $A^0(t)$, $B_p^0(t)$, $C^0_b(t)$ and $D^0_v(t)$ be the elapsed retrial time, elapsed service time of the priority customer, elapsed service time of the ordinary customer, elapsed service time of the G-Queue customer, and elapsed working vacation time respectively at time t . We introduce the random variable.

We define the limiting probability $Q_0(t) = P\{c(t) = 0, N(t) = 0\}$ and the probability densities.

We assume that the stability conditions are fulfilled in the sequence and so that we can set

$Q_0 = \log_{t \rightarrow \infty} Q_0(t)$; and limiting densities for $t \geq 0$, $x \geq 0$ and $n \geq 1$. $P_n(x) = \log_{t \rightarrow \infty} p_n(x, t)$;

$\pi_{1,n}(x) = \log_{t \rightarrow \infty} \pi_{1,n}(x, t)$; $\pi_{2,n}(x, y) = \log_{t \rightarrow \infty} \pi_{2,n}(x, y, t)$; $\pi_{b,n}(x) = \log_{t \rightarrow \infty} \pi_{b,n}(x, t)$ &

$Q_{v,n}(x) = \log_{t \rightarrow \infty} Q_{v,n}(x, t)$.

By the method of the supplementary variable, we easily the system of equilibrium equation:

$$\frac{d\pi_n(x)}{dx} = -(\lambda + \eta(x)) \pi_n(x) + \lambda^* \pi_{n-1}(x) (1 - \delta_n, 0), \quad n \geq 0 \quad (1)$$

$$\frac{dS_b(x)}{dx} = -(\lambda + \beta + \eta(x)) S_b(x) + \lambda^* S_{b-1}(x) (1 - \delta_n, 0), \quad n \geq 0 \quad (2)$$

$$\frac{dS_p(x)}{dx} = -(\lambda + \beta + \gamma(x)) S_p(x) + \lambda^* S_{p-1}(x) (1 - \delta_n, 0), \quad n \geq 0 \quad (3)$$

$$\frac{dR_n(x)}{dx} = -(\lambda^* + \varphi(x)) R_n(x) + \lambda^* R_{n-1}(x) (1 - \delta_n, 0), \quad n \geq 0 \quad (4)$$

$$\frac{dV_n(x)}{dx} = -(\lambda^* + \vartheta(x)) V_n(x) + \lambda^* V_{n-1}(x) (1 - \delta_n, 0), \quad n \geq 0 \quad (5)$$

$$[\lambda^* + \eta + \delta(1 - \delta_n, 0)] I_n = \int_0^\infty a_n(x) \eta(x) dx + \int_0^\infty S_b(x) \eta(x) dx + \int_0^\infty S_p(x) \gamma(x) dx + \int_0^\infty R_n(x) \varphi(x) dx + \int_0^\infty V_n(x) \vartheta(x) dx, \quad n \geq 0 \quad (6)$$

The steady-state boundary conditions are at $x = 0$ and $y = 0$.

$$H_{1,n}(0) = \int_0^\infty H_{2,n}(x) \eta(x) dx + (1-r) \int_0^\infty H_{4,n}(y) v_4(y) dy + \int_0^\infty H_{5,n}(y) v_5(y) dy + \int_0^\infty H_{6,n}(y) v_6(y) dy + \int_0^\infty H_{7,n}(x) v_7(x) dx, \quad n \geq 1 \quad (7)$$

$$H_{2,n}(0) = \lambda \int_0^\infty H_{1,n}(x) dx, \quad n \geq 1 \quad (8)$$

$$H_{3,n}(0) = \lambda \int_0^\infty H_{4,n}(y) dy, \quad n \geq 0 \quad (9)$$

$$H_{4,0}(0) = \left(\int_0^\infty H_{1,x} v_1(x) dx + \vartheta \int_0^\infty H_{6,0}(x) dx + \lambda p_0 \right), \quad n = 0 \quad (10)$$

$$H_{4,n}(0) = \left(\int_0^\infty H_{1,n+1}(x)dx + \lambda \int_0^\infty H_{1,n}(x)dx + \vartheta \int_0^\infty H_{6,n}(x)dx \right), n \geq 1 \dots (11)$$

$$H_{5,n}(0) = r \int_0^\infty H_{4,n}(y) v_4(y) dy, n \geq 0 \dots (12)$$

$$H_{6,n}(0) = \begin{cases} (\lambda + \lambda_1)Q, & n=0 \\ 0, & n \geq 1 \end{cases} \dots (13)$$

$$H_{7,n}(0) = \alpha \left(\int_0^\infty H_{2,n}(x)dx + \int_0^\infty \int_0^\infty H_{3,n}(x,y)dx dy + \int_0^\infty H_{4,n}(y)dy + \int_0^\infty H_{5,n}(y)dy \right), n \geq 0 \dots (14)$$

The system normalized condition is as follows:

$$P_0 + Q_0 + \sum_0^\infty H_{1,n}(y)dx + \sum_0^\infty \left(\int_0^\infty H_{2,n}(x)dx + \int_0^\infty \int_0^\infty H_{3,n}(x,y)dx dy + \int_0^\infty H_{4,n}(y)dy + \int_0^\infty H_{5,n}(y)dy + \int_0^\infty H_{6,n}(x)dx + \int_0^\infty H_{7,n}(x)dx \right) = 1.$$

PROBABILITY SOLUTIONS OF THE MODEL:

We define the generating functions for all of the states to solve the given systems of equations:

$$H_i(x,z) = \sum_0^\infty H_{i,n}(x)z^n; H_i(0,z) = \sum_{n=0}^\infty H_{i,n}(0)z^n;$$

Multiplying by z^n and summing over n to get generating functions for equations (1) – (14).

$$H_1(x,z) = H_1(0,z) [1-A(x)]e^{-(\lambda+\lambda_1)x} \dots (15)$$

$$H_2(x,z) = H_2(0,z)[1-S_b(x)]e^{-p(z)x} \dots (16)$$

$$H_3(x,y,z) = H_3(0,y,z) [1-S_b(x)]e^{-p(z)x} \dots (17)$$

$$H_4(y,z) = H_4(0,z) [1-S_p(x)] e^{-B(z)y} \dots (18)$$

$$H_5(y,z) = H_5(0,z) [1-S_p(x)] e^{-B(z)y} \dots (19)$$

$$H_6(x,z) = H_6(0,z) [1-R_p(x)] e^{-C(z)x} \dots (20)$$

$$H_7(x,z) = H_7(0,z) [1-V_b(x)] e^{-D(z)x} \dots (21)$$

Where $p(z) = (\alpha + \lambda(1-z))$, $B(z) = (p(z) + \lambda_1(1-S_p^*(p(z))))$,

$$D(z) = (\vartheta + \lambda_1(1-z) \text{ and } \lambda(z) = \lambda_1(1-z)$$

Using, the equations (1)-(14) in (15)-(21), we obtain as follows:

$$H_1(0,z) = H_2(0,z)S_b^*(p(z)) + (1-r)H_4(0,z)S_b^*(B(z)) + H_5(0,z)S_b^*(B(z)) + H_6(0,z)R^*(C(z)) \lambda(z) + H_7(0,z)V_b^*(D(z)) - (\lambda + \lambda_1 + P)Q_0.$$

To make some manipulations, we get the partial generating functions for all the states.

CHARACTERISTICS OF THE MODEL-I

1. To find the average (expected) number of customers in the system (L_s)

By definition of expected value

$$\begin{aligned} L_s &= \sum_{n=0}^\infty n P_n \\ &= \sum_{n=0}^\infty n \left(\frac{\lambda}{\mu} \right)^n \left(1 - \frac{\lambda}{\mu} \right) \\ &= \frac{\lambda}{\mu} \left(1 - \frac{\lambda}{\mu} \right) \sum_{n=1}^\infty n \left(\frac{\lambda}{\mu} \right)^{n-1} \\ &= \frac{\lambda}{\mu} \left(1 - \frac{\lambda}{\mu} \right) \left[1 + 2 \frac{\lambda}{\mu} + 3 \left(\frac{\lambda}{\mu} \right)^2 + \dots \right] \\ &= \frac{\lambda}{\mu} \left(1 - \frac{\lambda}{\mu} \right) \left(1 - \frac{\lambda}{\mu} \right)^{-2} \\ &= \frac{\frac{\lambda}{\mu}}{1 - \frac{\lambda}{\mu}} \\ &= \frac{\lambda}{\mu - \lambda} \end{aligned}$$

2. To find the expected (average) number of customers in the queue (L_q) (or) average length of queue:

If the number of customers in the system is n , then the number of customer's in the queue is $(n-1)$. (excluding one being serviced).

$$\begin{aligned} L_q &= \sum_{n=1}^\infty (n-1) P_n \\ &= \sum_{n=1}^\infty n P_n - \sum_{n=1}^\infty P_n \\ &= \sum_{n=1}^\infty n P_n - (\sum_{n=1}^\infty P_n - P_0) \\ &= \sum_{n=1}^\infty n P_n - (1 - P_0) \\ &= L_s - (1 - P_0) = \frac{\lambda^2}{\mu(\mu - \lambda)} \end{aligned}$$

3. To find the expected number of customers in nonempty queues (L_n):

Since the queue is non empty,

$$\begin{aligned}
 &= \frac{E(n-1)}{p(n-1>0)} = \frac{E(n-1)}{p(n>1)} \\
 &= \frac{E(n-1)}{\sum_{n=2}^{\infty} P_n} \\
 &= \frac{1}{\mu(\mu-\lambda)} \cdot \frac{1}{\left(\frac{\lambda^2}{\mu}\right)\left(1-\frac{\lambda}{\mu}\right) \sum_{n=0}^{\infty} \left(\frac{\lambda}{\mu}\right)^n} \text{ (Using Binomial Summation)} \\
 &= \frac{\mu}{\mu-\lambda}
 \end{aligned}$$

4. To find the probability that the number of customers in the system greater than or equal to k:

$$\begin{aligned}
 P(n \geq k) &= \sum_{n=k}^{\infty} P_n = \sum_{n=k}^{\infty} \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right) \\
 &= \left(\frac{\lambda}{\mu}\right)^k \left(1 - \frac{\lambda}{\mu}\right) \sum_{n=k}^{\infty} \left(\frac{\lambda}{\mu}\right)^{n-k} \\
 &= \left(\frac{\lambda}{\mu}\right)^k \left(1 - \frac{\lambda}{\mu}\right) \sum_{n=0}^{\infty} \left(\frac{\lambda}{\mu}\right)^n \\
 &= \left(\frac{\lambda}{\mu}\right)^k
 \end{aligned}$$

5. Probability density function of the waiting time and waiting time distribution function of the queue:

Let T_q be the random variable that represents the waiting time of a customer in the queue.

Let the probability density function of T_q be $f_q(w)$.

Let $f_q(w/n)$ be the density function of T_q given that there are n customers in the system (or) customers are waiting for service and one customer being served.

Then $f_q(w) \sum_{n=1}^{\infty} f_q(w/n) \rho_n$

$$\begin{aligned}
 \therefore f_q(w) &= \sum_{n=1}^{\infty} \frac{\mu^n}{(n-1)!} e^{-\mu w} w^{n-1} \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)^{n-1} \\
 &= \frac{\lambda}{\mu} (\mu - \lambda) e^{-\mu w} e^{\lambda w} \\
 &= \frac{\lambda}{\mu} (\mu - \lambda) e^{-(\mu - \lambda)w} ; w > 0 \\
 &= \mu \rho (1 - \rho) e^{-\mu(1-\rho)w} ; w > 0
 \end{aligned}$$

The distribution function $W_q(w)$ is given by

$$W_q(w) = 1 - \rho ; w = 0 = \Pr = P_0 = 1 - \rho = 1 - \rho e^{-\mu(1-\rho)w} ; w > 0.$$

Average (expected) waiting time of a customer in the queue: (W_q)

$$\begin{aligned}
 W_q = E(T_q) &= \int_0^{\infty} w \cdot \frac{\lambda}{\mu} (\mu - \lambda) e^{-(\mu - \lambda)w} dw \\
 &= \frac{\lambda}{\mu} (\mu - \lambda) \int_0^{\infty} w \cdot e^{-(\mu - \lambda)w} dw \\
 &= \frac{\lambda}{\mu(\mu - \lambda)}
 \end{aligned}$$

The present investigation incorporates the following features simultaneously:

- Preemptive priority retrial queueing: Priority customers are given immediate preference over ordinary customers, and ordinary customers may be interrupted when a priority customer arrives.
- Negative customers: Negative customers are considered in the system, which can remove customers from the queueing system and thereby influence system performance.
- G-Queue: The model incorporates a G-Queue mechanism, allowing customers with different effects on the system to be considered.
- Bernoulli vacations: The server may take vacations according to a Bernoulli vacation policy, introducing additional server interruptions into the system.
- Retrial mechanism: Customers who cannot obtain service immediately enter an orbit and retry for service after a random time.
- Supplementary variable technique: This technique is employed to derive the probability generating functions (PGFs) and obtain the steady-state system characteristics.

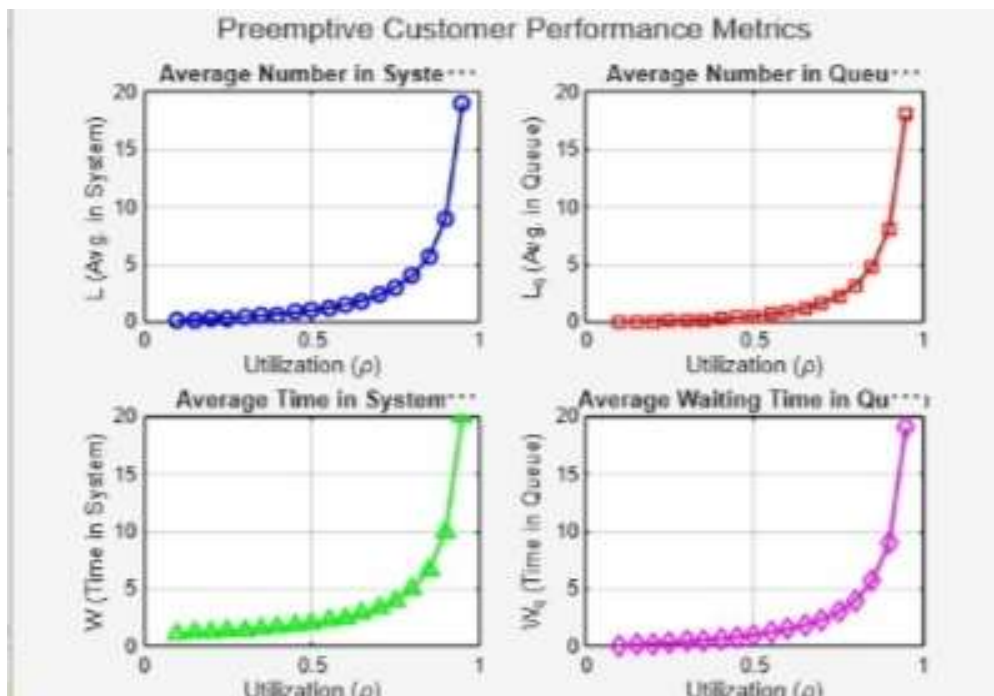
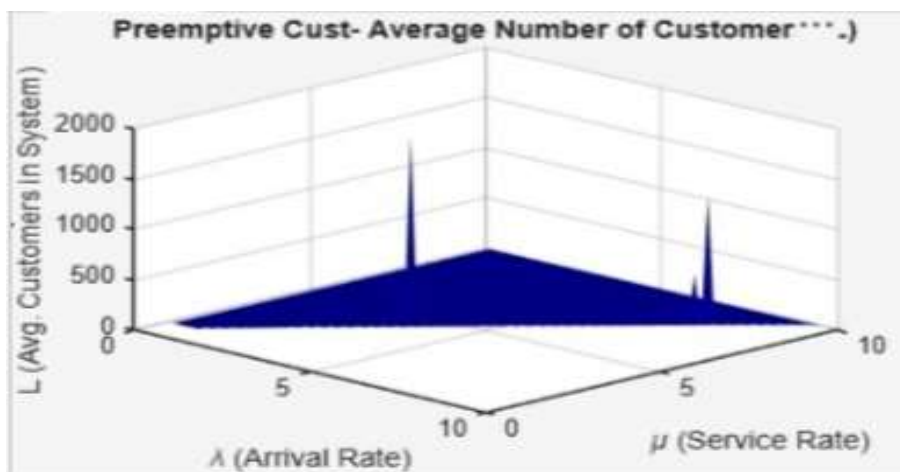
Performance measures: Important performance measures, including the mean number of customers in the orbit and the mean number of customers in the system, are derived.

Stochastic decomposition: A stochastic decomposition property is established to represent the system characteristics in terms of suitable component systems.

Numerical validation: The analytical results are illustrated and validated through numerical examples.

Conclusions:

In this paper, we have examined a preemptive priority retrial queueing system with the G-Queue customer, and the Bernoulli vacation. Using the method of supplementary variable technique, the PGFs for the numbers of customers in the system when it is free, busy with priority customer, busy with preemptive priority customer, busy with an ordinary customer, on vacations. Some important system performance measures and stochastic decomposition law are also discussed. The explicit expressions for the average queue length of orbit and system have been obtained. Finally, the analytical results are validated with the help of numerical illustrations.



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