

A Plithogenic Decision Making Framework For Selecting Adolescent Mental Health Intervention Strategies Using Plithogenic Hesitant Fuzzy Soft Matrices

J. Jayasudha¹, R.Nishanthi²

¹ Associate Professor, Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi-642001, Tamilnadu, India. E-mail : jayasudhaj@ngmc.org

² Assistant Professor, Department of Mathematics, Dr.Mahalingam college of Engineering and Technology, Pollachi-642003, Tamilnadu, India.E-mail: raghunisha94@gmail.com

Abstract

In this paper we introduce and study the concept of Plithogenic hesitant fuzzy soft matrices (P-HFSM) and their operations which are more functional to make theoretical studies in the Plithogenic soft set theory. Further, we develop a multi attribute decision making problems using TOPSIS method for Plithogenic hesitant fuzzy soft sets.

Keywords: Hesitant Fuzzy Soft Set, Plithogenic Hesitant Fuzzy Soft Set, Hesitant Fuzzy Soft matrix, Plithogenic Hesitant Fuzzy Soft matrix, Decision making using TOPSIS.

1 Introduction

In 1965, Zadeh [1] created the concept of fuzzy sets, which addresses probabilistic uncertainty and has found application in decision-making problems. Molodtsov first proposed soft set theory [5] in 1999 as a generic mathematical tool for handling uncertainties that classical mathematical methods are unable to handle. Molodtsov has demonstrated numerous uses of this theory to address numerous real-world issues in the fields of engineering, economics, social science, medicine, and more. Later on, Maji et al. [6] defined fuzzy soft sets. Maji et al. introduced many operations on soft sets and explored theoretical features of soft sets in [7]. Since then, several researchers have extended and applied the concept of soft set theory, including Aktas and Cagman [9], Roy and Maji [10], Yang et al. [11], Majumdar and Samanta [12], Xiao et al. [13], with a variety of successful outcomes. Both truth-membership and falsity-membership must be taken into account in some real-world situations involving expert systems, belief systems, information fusion, and other fields in order to represent an item in an ambiguous, uncertain context. Neither the fuzzy sets nor the interval-valued fuzzy sets [3] are suitable in this situation. Atanassov's intuitionistic fuzzy sets [4] make sense in this type of situation. Intuitionistic fuzzy sets can only manage incomplete data that considers both truth-membership (or simply membership) and falsity-membership (or non-membership) values. The unclear and conflicting information present in belief systems is not addressed by it. In order to resolve situations involving inconsistent, imprecise, and ambiguous data, Smarandache [8] devised a mathematical tool called the neutrosophic set. Neutrosophic sets deal with three components such as membership, indeterminacy and non-membership.

Recently, Torra [14] and Torra and Narukawa [15] introduced the idea of a hesitant fuzzy set to address the challenge of determining the degree of membership of an element in a set. This new fuzzy set extension can handle situations where our hesitation among a few different values causes the difficulty in determining the membership degree, rather than a margin of error (as in intuitionistic or interval-valued fuzzy sets) or a specified possibility distribution of the possible values (as in type-2 fuzzy sets [2]). Therefore, compared to the fuzzy set and its several classical extensions, the hesitant fuzzy set can more properly reflect people's uncertainty when expressing their preferences for objects. In 2011, Xu and Xia [16] introduced the idea of the hesitant fuzzy element (HFE), which is a straightforward and useful tool for expressing the hesitating preferences of decision makers during the decision-making process. Since then, our research team has conducted extensive studies on clustering analysis, aggregation, distance, similarity and correlation metrics, and decision-making with uncertain fuzzy data.

One of the most important factors of decision-making in daily life is accuracy. In order to improve accuracy, Smarandache's plithogeny [18] theory is defined as a more general way to represent sets of various types, including intuitionistic, fuzzy, crisp, and neutrosophic. Plithogenic sets comprise degrees of appurtenance and contradiction in relation to merely the dominant characteristic. In 2017, Smarandache described a plithogenic set P as a set whose elements are specified by one or more attributes, each of which can have numerous values.. A plithogenic set is the foundation for all plithogenic functions (plithogenic set, plithogenic probability, plithogenic statistics, and plithogenic logic), which are defined by a single appurtenance in a plithogenic set. Crisp sets have a single value (membership), fuzzy sets have two characteristics (membership and non-

membership), intuitionistic sets have three characteristics (membership, indeterminate, and non-membership), and plithogenic sets have four or more characteristics.

2 Preliminaries

In this section, we will review some of the fundamental definitions required to read this paper.

Definition 2.1 [5]

A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U . In other words, the soft set is a parameterized collection of subsets of the set U . Every set $F(\varepsilon)$, $\varepsilon \in E$, belonging to this family can be regarded as either the set of ε - elements of the soft set (F, E) , or as the set of ε - approximate elements of the soft set.

Definition 2.2 [6]

A fuzzy soft set over U is a pair (F, A) , where $F : A \rightarrow P(U)$ is a mapping from A into $P(U)$.

Definition 2.3 [15]

If X is a fixed set, then a Hesitant Fuzzy Set (HFS) on X is defined in terms of a function $h_M(x)$ that returns a subset of $[0, 1]$, when it is applied to X . The following mathematical expression can be used to express this : M is defined as $\{ \langle x, h_M(x) \rangle \mid x \in X \}$, where $h_M(x)$ is a set of values in $[0, 1]$ that represent the various membership degrees of the element $x \in X$ to the set M . We refer to $h = h_M(x)$ as a hesitant fuzzy element (HFE) for simplicity, and H is the set of all HFEs.

Definition 2.4 [17]

Let U be a universal set and E be set of parameters. Let $HF(U)$ denotes the set of all hesitant fuzzy sets defined over U . A pair (F, E) is a hesitant fuzzy soft set if $F(e) \in HF(U)$ for every e in E .

Definition 2.5 [18]

Let U be the discourse universe and P be the non-empty set of elements, such that $P \subseteq U$. Then, a is an attribute (generally multidimensional), V is the range of the attribute values, d is the degree to which each element x 's attribute value belongs to the set P in relation to a given set of criteria ($x \in P$), and c is the degree of contradiction between attribute values. Then, (P, a, V, d, c) is called a plithogenic set.

Definition 2.6 [19]

Assume that U is a discourse universe and that $(U)^Z$ is the z -power set of U such that

$z = C$ (Power set of U),

$z = F$ (The set of all fuzzy sets of U),

$z = IF$ (The set of all intuitionistic fuzzy sets of U), or

$z = N$ (The set of all neutrosophic sets of U).

Let $a_1, a_2, \dots, a_n$, for $n \geq 1$, be n distinct attributes, where corresponding attribute values $V_1, V_2, \dots, V_n$, with $V_i \cap V_j = \emptyset$, for $j \neq i$, and $i, j \in \{1, 2, \dots, n\}$. Suppose $V_i = \{v_{i1}, v_{i2}, \dots, v_{imi}\}$ and let $Y = V_1 \times V_2 \times \dots \times V_n$. Let $c(D_i, v_{ij})$, $i \in \{1, 2, \dots, n\}$, $j \in \{1, 2, \dots, m_i\}$ be the attribute value contradiction degree function such that $c_i: V_i \times V_i \rightarrow [0, 1]$, and let $D = (D_1, D_2, \dots, D_n)$ the dominant attribute element of $V_i, \forall i$. Then the pair (F_P^Z, Y) , where: $F_P^Z: Y \rightarrow [0, 1]_D \times (U)^Z$ is called a Plithogenic Soft Set (P-SS In short) over U .

Definition 2.7 [20]

Let U be a universe of discourse, $(U)^H$ the power set of U such that H is the set of all Hesitant fuzzy soft sets. Let $a_1, a_2, \dots, a_n$, for $n \geq 1$, be n distinct attributes, each of which has corresponding attribute values are respectively $V_1, V_2, \dots, V_n$, with $V_i \cap V_j = \emptyset$, for $j \neq i$, and $i, j \in \{1, 2, \dots, n\}$. Assume $V_i = \{v_{i1}, v_{i2}, \dots, v_{imi}\}$ and $\mathcal{H} = V_1 \times V_2 \times \dots \times V_n$. Let $D = (D_1, D_2, \dots, D_n)$ be the dominant attribute element of $V_i, \forall i$ and $c(D_i, v_{ij})$, $i \in \{1, 2, \dots, n\}$, $j \in \{1, 2, \dots, m_i\}$ the attribute value contradiction degree function $c_i: V_i \times V_i \rightarrow [0, 1]$. Then $(F_P^H, \mathcal{H})$, where: $F_P^H: \mathcal{H} \rightarrow [0, 1]_D \times (U)^H$ is called a Plithogenic hesitant fuzzy soft set (P-HFSS In short) over U .

Definition 2.8 [17]

HFSS set can be represented by HFSM. If $X = \{x_1, x_2, \dots, x_m\}$ and $E = \{e_1, e_2, \dots, e_n\}$, then HFSM could be defined as $F = (f_{ij})_{m \times n}$, where $f_{ij} = h_M^k(x_i, e_j)$, $i = 1, 2, \dots, m$ & $j = 1, 2, \dots, n$. Here $k > 0$ be the number of values which may be different for different HFE.

Definition 2.9 [17]

Let $\omega = (\tilde{F}_A, E)$ and $\varpi = (\tilde{G}_A, E)$ be two hesitant fuzzy soft sets over X . Let 'd' be a mapping that $d : HFSS(X) \times HFSS(X) \rightarrow R^+ \cup \{0\}$ (where $R^+ \cup \{0\}$ represents a set of non-negative real numbers). Then distance $d(\omega, \varpi)$ between hesitant fuzzy soft sets ω and ϖ satisfies the following properties (1)-(3):

(1) $d(\omega, \varpi) \geq 0$; (2) $d(\omega, \varpi) = d(\varpi, \omega)$; (3) $d(\omega, \varpi) = 0$ iff $\omega = \varpi$.

Definition 2.10 [17]

Let $\omega = (\tilde{F}_A, E) = [a_{ij}]_{m \times n}$ and $\varpi = (\tilde{G}_A, E) = [b_{ij}]_{m \times n}$ be two HFSSs over X . The Hamming distance $d_{HFSS}^H(\omega, \varpi)$ and Euclidean distance $d_{HFSS}^E(\omega, \varpi)$ between ω and ϖ are defined as follows:

$$d_{\text{HFSS}}^H(\omega, \varpi) = \sum_{i=1}^m \sum_{j=1}^n \sum_k \frac{|h_{F_A}^k(x_i, e_j) - h_{G_A}^k(x_i, e_j)|}{k}$$

$$d_{\text{HFSS}}^E(\omega, \varpi) = \left(\sum_{i=1}^m \sum_{j=1}^n \sum_k \frac{\{ |h_{F_A}^k(x_i, e_j) - h_{G_A}^k(x_i, e_j)| \}^2}{k} \right)^{\frac{1}{2}}.$$

Definition 2.11 [17]

Score of a hesitant fuzzy soft set $\omega = (\tilde{F}_A, E)$ can be defined by:

$$\text{Score}(\omega) = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n \left(\frac{1}{k} \sum_k h_{\omega}^k(x_i, e_j) \right), e \in E, x \in X.$$

The score of a parameter $e \in E$ is defined as follows:

$$\text{Score}_{\{e\}}(\omega) = \frac{1}{m} \sum_{i=1}^m \left(\frac{1}{k} \sum_k h_{\omega}^k(x_i, e) \right), x \in X.$$

3 Plithogenic Hesitant Fuzzy Soft Matrices

In this section we define the Plithogenic hesitant fuzzy soft matrix and some of its types.

Definition 3.1

Let $X = \{x_1, x_2, \dots, x_m\}$ and $E = \{e_1, e_2, \dots, e_n\}$. Then the PHFSM is defined as $M(F_P^H) = (f_{ij})_{m \times n}$, where $f_{ij} = F_P^H(x_i, e_j)$, $i = 1, 2, \dots, m$ & $j = 1, 2, \dots, n$.

Example 3.2

Choosing the right college is one of the most important decisions in a student's academic and professional journey. The selection process is inherently complex, as it involves multiple conflicting criteria such as Infrastructure, Quality of education, Placement and each attribute has multiple attribute values with multiple memberships. To simplify the explanation, let U represents the set of colleges where $U = \{C_1, C_2, C_3\}$. Let the distinct attributes $a_1 = \text{Infrastructure}$, $a_2 = \text{Quality of education}$, $a_3 = \text{Placement}$ and their corresponding attribute values are $\text{Infrastructure} \equiv A_1 = \{\text{Good}, \text{Not bad}, \text{Bad}\}$, $\text{Quality of education} \equiv A_2 = \{\text{Above avg}, \text{Avg}, \text{Below avg}\}$, $\text{Placement} \equiv A_3 = \{\text{High}, \text{Medium}, \text{Low}\}$, where $D = (\text{Good}, \text{Above avg}, \text{High})$ denotes the dominant attribute values.

Let $E \subseteq \mathcal{H}$ such that $E = \{\varepsilon_1 = (\text{not bad}, \text{above avg}, \text{high}), \varepsilon_2 = (\text{good}, \text{avg}, \text{medium}), \varepsilon_3 = (\text{not bad}, \text{avg}, \text{medium})\}$.

Define a function $F_P^H: E \rightarrow [0, 1]_D \times (U)^H$ such that

$$F_P^H(\varepsilon_1) = \left\{ \frac{(C_1, (0.3, 0, 0)_D)}{\{(0.6, 0.2), (0.7, 0.1, 0.3), (0.5, 0.4)\}}, \frac{(C_2, (0.3, 0, 0)_D)}{\{(0.2, 0.3, 0.4), (0.1, 0.2), (0.7, 0.9, 1)\}}, \frac{(C_3, (0.3, 0, 0)_D)}{\{(0.1, 0.6), (0.7, 0.9), (0.2, 0.7)\}} \right\}$$

$$F_P^H(\varepsilon_2) = \left\{ \frac{(C_1, (0, 0.6, 0.8)_D)}{\{(0.1, 0.4), (0.7, 0.2, 0.5), (0.9, 1)\}}, \frac{(C_2, (0, 0.6, 0.8)_D)}{\{(0.3, 0.8), (0.4, 0.7, 0.9), (0.1, 0.6, 1)\}}, \frac{(C_3, (0, 0.6, 0.8)_D)}{\{(0.5, 0.8), (0.5, 0.6, 1), (0.2, 0.4)\}} \right\}$$

$$F_P^H(\varepsilon_3) = \left\{ \frac{(C_1, (0.5, 0.7, 0.6)_D)}{\{(0.1, 0.2), (0.5, 0.6, 0.7), (0.1)\}}, \frac{(C, (0.5, 0.7, 0.6)_D)}{\{(0.4, 0.8), (0.9, 1), (0.2, 0.4, 0.6)\}}, \frac{(C_3, (0.5, 0.7, 0.6)_D)}{\{(0, 0.5, 1), (0, 0.6), (0.2, 0.9)\}} \right\}$$

Then we have the P- HFSS as follows :

$$(F_P^H, E) = \left(\begin{array}{c} \varepsilon_1, \left\{ \begin{array}{c} \frac{(C_1, (0.3, 0, 0)_D)}{\{(0.6, 0.2), (0.7, 0.1, 0.3), (0.5, 0.4)\}}, \\ \frac{(C_2, (0.3, 0, 0)_D)}{\{(0.2, 0.3, 0.4), (0.1, 0.2), (0.7, 0.9, 1)\}}, \\ \frac{(C_3, (0.3, 0, 0)_D)}{\{(0.1, 0.6), (0.7, 0.9), (0.2, 0.7)\}} \end{array} \right\} \\ \varepsilon_2, \left\{ \begin{array}{c} \frac{(C_1, (0, 0.6, 0.8)_D)}{\{(0.1, 0.4), (0.7, 0.2, 0.5), (0.9, 1)\}}, \\ \frac{(C_2, (0, 0.6, 0.8)_D)}{\{(0.3, 0.8), (0.4, 0.7, 0.9), (0.1, 0.6, 1)\}}, \\ \frac{(C_3, (0, 0.6, 0.8)_D)}{\{(0.5, 0.8), (0.5, 0.6, 1), (0.2, 0.4)\}} \end{array} \right\} \\ \varepsilon_3, \left\{ \begin{array}{c} \frac{(C_1, (0.5, 0.7, 0.6)_D)}{\{(0.1, 0.2), (0.5, 0.6, 0.7), (0.1)\}}, \\ \frac{(C, (0.5, 0.7, 0.6)_D)}{\{(0.4, 0.8), (0.9, 1), (0.2, 0.4, 0.6)\}}, \\ \frac{(C_3, (0.5, 0.7, 0.6)_D)}{\{(0, 0.5, 1), (0, 0.6), (0.2, 0.9)\}} \end{array} \right\} \end{array} \right)$$

The Plithogenic hesitant fuzzy soft matrix $M(F_P^H, A)$ of the given P-HFSS as follows:

$$[f_{ij}] = \begin{bmatrix} F_P^H(C_1, \varepsilon_1) & F_P^H(C_1, \varepsilon_2) & F_P^H(C_1, \varepsilon_3) \\ F_P^H(C_2, \varepsilon_1) & F_P^H(C_2, \varepsilon_2) & F_P^H(C_2, \varepsilon_3) \\ F_P^H(C_3, \varepsilon_1) & F_P^H(C_3, \varepsilon_2) & F_P^H(C_3, \varepsilon_3) \end{bmatrix}$$

Definition 3.3

Let $M(F_P^H, A) = [f_{ij}]$ be a P-HFSM over a P-HFSS (F_P^H, A) with respect to the universe U . Then the compliment of the P-HFSM $M(F_P^H, A)$ is denoted by $[M(F_P^H, A)]^C$, where

$$[M(F_P^H, A)]^C = [f_{ij}]^C = \begin{bmatrix} F_P^H(C_1, \varepsilon_1)^C & F_P^H(C_1, \varepsilon_2)^C & F_P^H(C_1, \varepsilon_3)^C \\ F_P^H(C_2, \varepsilon_1)^C & F_P^H(C_2, \varepsilon_2)^C & F_P^H(C_2, \varepsilon_3)^C \\ F_P^H(C_3, \varepsilon_1)^C & F_P^H(C_3, \varepsilon_2)^C & F_P^H(C_3, \varepsilon_3)^C \end{bmatrix}.$$

Example 3.4

Reconsider Example 3.2. The compliment of the P-HFSM $M(F_P^H, A)$ is defined by,

$$[f_{ij}]^C = \begin{bmatrix} \{(0.8, 0.4), (0.7, 0.9, 0.3), (0.6, 0.5)\} & \{(0.6, 0.9), (0.5, 0.8, 0.3), (0, 0.1)\} & \{(0.8, 0.9), (0.3, 0.4, 0.5), ((0.9))\} \\ \{(0.6, 0.7, 0.8), (0.8, 0.9), (0, 0.1, 0.2)\} & \{(0.2, 0.7), (0.1, 0.3, 0.6), (0, 0.4, 0.9)\} & \{(0.2, 0.6), (0, 0.1), (0.4, 0.6, 0.5)\} \\ \{(0.4, 0.9), (0.1, 0.3), (0.3, 0.8)\} & \{(0.2, 0.5), (0, 0.4, 0.5), (0.6, 0.8)\} & \{(0.5, 1), (0.4, 1), (0.1, 0.8)\} \end{bmatrix}$$

Definition 3.5

The union of two P-HFSMs $M(F_P^H, A) = [f_{ij}]$ and $M(G_P^H, B) = [g_{ij}]$ over the PHFSSs (F_P^H, A) and (G_P^H, B) is denoted by $M(F_P^H, A) \vee_P^H M(G_P^H, B)$ is a PHFSM $M(H_P^H, C) = [h_{ij}]$ and is defined by $[h_{ij}] = [f_{ij}] \cup [g_{ij}]$.

Definition 3.6

The intersection of two P-HFSMs $M(F_P^H, A) = [f_{ij}]$ and $M(G_P^H, B) = [g_{ij}]$ over the P-HFSSs (F_P^H, A) and (G_P^H, B) is denoted by $M(F_P^H, A) \wedge_P^H M(G_P^H, B)$ is a PHFSM $M(H_P^H, C) = [h_{ij}]$ and is defined by $[h_{ij}] = [f_{ij}] \cap [g_{ij}]$.

Definition 3.7

The two P-HFSMs $M(F_P^H, A) = [f_{ij}]$ and $M(G_P^H, B) = [g_{ij}]$ are said to be equal matrices if $[f_{ij}] = [g_{ij}]$ for all i and j .

Proposition 3.8

Let $[f_{ij}]$, $[g_{ij}]$ and $[h_{ij}]$ are the P-HFSMs. Then

- (i) $[f_{ij}] = [g_{ij}]$ and $[g_{ij}] = [h_{ij}] \Rightarrow [f_{ij}] = [h_{ij}]$
- (ii) $[f_{ij}] \subseteq [g_{ij}]$ and $[g_{ij}] \subseteq [f_{ij}] \Rightarrow [f_{ij}] = [g_{ij}]$
- (iii) $([f_{ij}])^C = [f_{ij}]$

4 Distance measures of Plithogenic Hesitant Fuzzy Soft Sets

In this section, we define the distance measure of Plithogenic Hesitant Fuzzy Soft Sets.

Definition 4.1

The distance measure $d((F_P^H, A), (G_P^H, B))$ between the P-HFSSs (F_P^H, A) and (G_P^H, B) satisfies the following properties:

- (i) $d((F_P^H, A), (G_P^H, B)) \geq 0$
- (ii) $d((F_P^H, A), (G_P^H, B)) = d((G_P^H, B), (F_P^H, A))$
- (iii) $d((F_P^H, A), (G_P^H, B)) = 0$ if and only if $(F_P^H, A) = (G_P^H, B)$.

Definition 4.2

Let $M = (F_P^H, E)$ and $N = (G_P^H, E)$ be two P-HFSSs over the universal set U . The Hamming distance $d_{PHFSS}^H(M, N)$ between M and N are defined as follows :

$$d_{PHFSS}^H(M, N) = \sum_{i=1}^m \sum_{j=1}^n \sum_k \frac{|F_P^H(C_i, \varepsilon_j) - G_P^H(C_i, \varepsilon_j)|}{k}.$$

Definition 4.3

Let $M = (F_P^H, E)$ and $N = (G_P^H, E)$ be two P-HFSSs over the universal set U . The Euclidean distance $d_{PHFSS}^E(M, N)$ between M and N are defined as follows :

$$d_{PHFSS}^E(M, N) = \left(\sum_{i=1}^m \sum_{j=1}^n \sum_k \frac{\{ |F_P^H(C_i, \varepsilon_j) - G_P^H(C_i, \varepsilon_j)| \}^2}{k} \right)^{\frac{1}{2}}$$

5 An Plithogenic TOPSIS based MADM framework for Prioritizing Mental Health Intervention Strategies

TOPSIS is a mathematical MADM method that is used to solve decision-making problems in many different domains and has demonstrated impressive outcomes in several research.

In this study, we propose a Plithogenic MADM method using Euclidean and Hamming distance. The approach begins with the aggregation of expert evaluations expressed in Plithogenic form, followed by normalization and weighting of criteria. A weighted normalized decision matrix is constructed, from which the positive ideal and negative ideal solutions are determined. Distances between each alternative and a closeness coefficient are derived to rank the alternatives. This framework effectively integrates the expressive power of Plithogenic sets with the computational efficiency of TOPSIS.

This can be expressed in the following algorithm:

Step 1: Aggregation of expert evaluations with respect to the alternatives L_i under the attribute A_j in terms of P-HFSM.

Step 2: Apply TOPSIS vector normalization formula to normalize the decision matrix.

$$P_{ij} = \frac{v_{ij}}{\sqrt{\sum_{i=1}^m v_{ij}^2}}$$

where refers v_{ij} to the value in decision matrix, and Plithogenic adjustment formula $CP_{ij} = (1 - c) P_{ij} + c (1 - P_{ij})$.

Step 3: Construct the weighted decision matrix as:

$$Q_{ij} = w_{ij} * CP_{ij}$$

Step 4: Calculate the positive & negative ideal solutions

$V^+ = (v_1^+, v_2^+, v_3^+, \dots)$ and $V^- = (v_1^-, v_2^-, v_3^-, \dots)$,

where

$$v_j^+ = \begin{cases} \max v_{ij} , & \text{for benefit criteria} \\ \min v_{ij} , & \text{for cost criteria} \end{cases}$$

$$v_j^- = \begin{cases} \min v_{ij} , & \text{for benefit criteria} \\ \max v_{ij} , & \text{for cost criteria} \end{cases}$$

Step 5: Compute the distance of every alternative using Euclidean distance and Hamming distance.

Step 6: Compute the closeness coefficient of every alternative $R_i = \frac{D_i^-}{D_i^+ + D_i^-}$.

Step 7: Rank the alternatives.

The above-described steps are presented graphically for better understanding in figure 1.

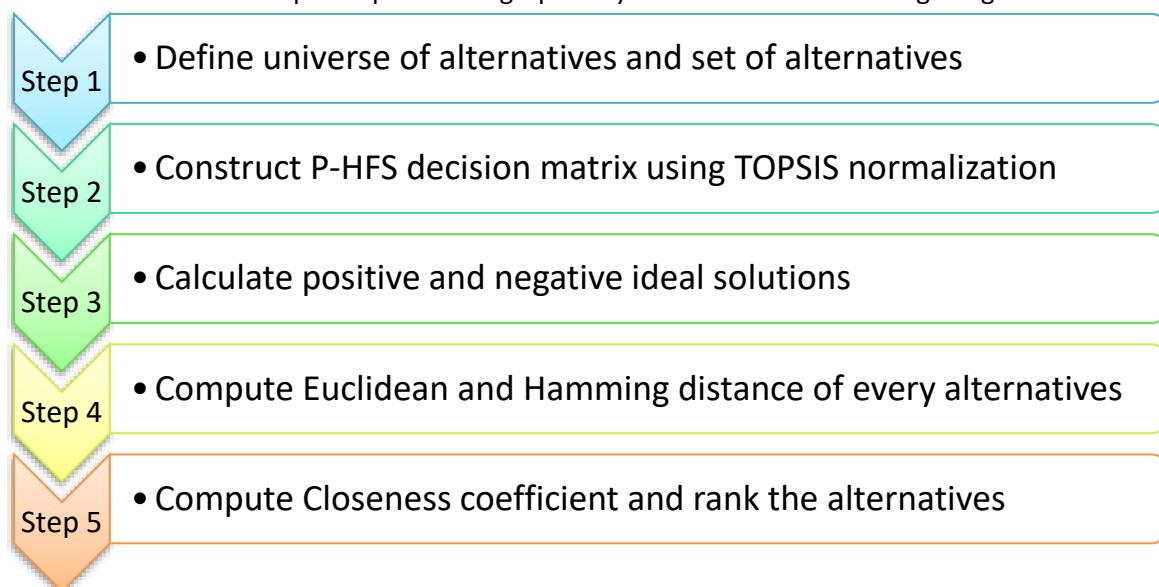


Figure 1

Case study: Adolescent Mental health and Preventive health care

Adolescent mental health is a significant public health concern. A government health department proposes implementing an integrated mental health assistance program for teenagers aged 13 to 19. Three specialists, a child and adolescent psychiatrist, a clinical psychologist, and a public health specialist, are assigned to assess three possible intervention strategies. The purpose is to select the most appropriate intervention technique based on its effectiveness, accessibility, affordability, acceptability among adolescents, and ability to preserve privacy.

Let the set of alternatives be $U = \{L_1, L_2, L_3\}$, here

L_1 = School-based mental health counselling and early screening,

L_2 = Digital mental health support using AI-assisted screening and tele-counselling, L_3 = Community-based mental health support involving families, peer groups, and healthcare professionals and

Attributes (Criteria) = $\{A_1, A_2, A_3, A_4, A_5\}$, where

A_1 = Effectiveness in improving adolescent mental well-being,

A_2 = Accessibility and availability of professional support

A_3 = Acceptability and willingness of adolescents to participate

A_4 = Cost of implementation and maintenance

A_5 = Privacy protection and confidentiality of adolescent information

with corresponding attribute values for $A_1 = \{High, Medium, Low\}$, $A_2 = \{High, Moderate, Low\}$, $A_3 = \{Excellent, Good, Poor\}$, $A_4 = \{Low, Medium, High\}$ and $A_5 = \{Good, Avg, poor\}$, where A_1, A_2, A_3 & A_5 are benefit criteria and A_4 is cost criteria.

Let the Dominant attribute value $D = \{High, High, Excellent, Low, Good\}$

The three experts provide their opinions based on their professional knowledge. Although intervention outcomes, accessibility, individual preferences, and privacy concerns can vary, their assessments may contain hesitation and uncertainty.

Step 1: Three experts were given the evaluation of this study as Plithogenic decision making matrix (row-alternatives & column-criteria) as shown in Tables 4-6.

Table-4

Expert-1	A_1	A_2	A_3	A_4	A_5
L_1	(0.1,0.2)	(0.1,0.3,0.5)	(0.4,0.5)	(0.5,0.6)	(0.7,0.9)
L_2	(0.2,0.3,0.4)	(0.1,0.2)	(0.7,0.9,1)	(0.3,0.4)	(0.6,0.7)
L_3	(0.1,0.6)	(0.7,0.9)	(0.2,0.7)	(0.8,0.9)	(0.7,0.9)

Table-5

Expert-2	A_1	A_2	A_3	A_4	A_5
L_1	1,0.4)	2,0.5,0.7)	1,0.9)	2,0.4)	9,1)
L_2	3,0.8)	4,0.7,0.9)	1,0.6,1)	6,0.7)	5,0.7,0.9)
L_3	5,0.8)	5,0.6,1)	2,0.4)	1,0.2,0.3)	9,1)

Table-6

Expert-3	A_1	A_2	A_3	A_4	A_5
L_1	2,0.4,0.6)	8,0.9)	5,1)	2,0.4)	6,0.7)
L_2	7,0.8,0.9)	6,0.7)	5,0.7,0.9)	3,0.5)	1,0.2)
L_3	3,0.4,0.5)	3,0.6)	5,0.6,0.7)	6,0.8,0.9)	6,0.9,1)

Step 2: We normalize the decision matrix to normalization matrix as follows.

$$[P_{ij}] = \begin{bmatrix} 0.9116 & 0.9805 & 0.5587 & 0.4310 & 0.6263 \\ 0.3133 & 0.2560 & 0.7007 & 0.5240 & 0.3914 \\ 0.2658 & 0.3943 & 0.4437 & 0.7347 & 0.6741 \end{bmatrix}$$

Let contradiction degrees according to the experts evaluation, for exact match=0, adjacent : 0.1 to 0.3=0.8, 0.4 to 0.7=0.5 or 0.6 & 0.6 to 0.9=0.2 and opposite=1. Suppose let the contradiction matrix,

$$[C_{ij}] = \begin{bmatrix} 0.8 & 0.6 & 0.6 & 0.5 & 0.2 \\ 0.2 & 0.5 & 0.2 & 0.5 & 0.6 \\ 0.6 & 0.2 & 0.5 & 0.2 & 0.2 \end{bmatrix}$$

Then contradiction-adjusted matrix,

$$[CP_{ij}] = \begin{bmatrix} 0.2530 & 0.4039 & 0.4882 & 0.5 & 0.5757 \\ 0.3879 & 0.5 & 0.6204 & 0.5 & 0.5217 \\ 0.5468 & 0.4365 & 0.5 & 0.6408 & 0.6044 \end{bmatrix}$$

Step 3: Compute the weighted normalized matrix by using the weight vectors

$$w = \{0.25, 0.25, 0.20, 0.15, 0.15\}.$$

$$[Q_{ij}] = \begin{bmatrix} 0.0632 & 0.1009 & 0.0976 & 0.0750 & 0.0863 \\ 0.0969 & 0.1250 & 0.1240 & 0.0750 & 0.0782 \\ 0.1367 & 0.1091 & 0.1 & 0.0961 & 0.0906 \end{bmatrix}$$

Step 4: Determine the positive & negative ideal solutions:

Positive ideal solution $V^+ = [0.1367 \quad 0.1250 \quad 0.1240 \quad 0.0750 \quad 0.0906]$

Negative ideal solution $V^- = [0.0632 \quad 0.1009 \quad 0.0976 \quad 0.0961 \quad 0.0782]$, where A_1, A_2, A_3 & A_5 are benefit criteria and A_4 is cost criteria.

Step 5: Case (i): Compute the distance of every alternative using Euclidean distance

$$D_i^+ = \left[\sum_j (v_{ij} - v_j^+)^2 \right]^{\frac{1}{2}} \text{ and}$$

$$D_i^- = \left[\sum_j (v_{ij} - v_j^-)^2 \right]^{\frac{1}{2}}$$

Case (ii): Compute the distance of every alternative using Hamming distance

$$D_i^+ = \left[\sum_j (v_{ij} - v_j^+)^2 \right] \text{ and}$$

$$D_i^- = \left[\sum_j (v_{ij} - v_j^-)^2 \right].$$

Step 6: The relative closeness of each alternative to the ideal solutions and rank of the alternatives in both the cases are shown in Table -5 & 6.

Table-7-Case(i)

Alternatives	Closeness values	Rank
L_1	0.0718	3
L_2	0.6228	2
L_3	0.8149	1

Table-7-Case(ii)

Alternatives	Closeness values	Rank
L_1	0.0102	3
L_2	0.5945	2
L_3	0.9632	1

The alternatives are ranked based on the highest value in the closeness values. Thus, in both the cases the most desirable alternative is L_3 .

6 Conclusion

This paper introduces the notion of Plithogenic hesitant fuzzy soft matrices (P-HFSM) based on P-HFSSs and some of its properties. Also, we developed MADM frameworks based on TOPSIS

method. The approach effectively integrates the expressive power of plithogenic theory with the computational efficiency of TOPSIS to address complex decision-making problems under uncertainty, hesitation and contradiction.

References

- [1] L.A. Zadeh, Fuzzy sets, *Information and Control*, vol. 8, 1965, pp. 338–353.
- [2] Dubois, D., Prade, H. *Fuzzy sets and systems: Theory and Applications*. Academic Press, New York (1980).
- [3] I. Turksen, Interval valued fuzzy sets based on normal forms, *Fuzzy Sets and Systems* 20 (1986) 191–210.
- [4] Krassimir T. Atanassov. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1):87–96, 1986.
- [5] D. Molodtsov, Soft set theory—first results, *Computers & Mathematics with Applications*, vol. 37, no. 4-5, pp. 19–31, 1999.
- [6] P.K. Maji, R. Biswas, and A.R. Roy, “Fuzzy Soft Sets”, *Journal of Fuzzy Mathematics*, vol. 9 (3), 2001, pp. 589–602.
- [7] P.K. Maji, R. Biswas, and A.R. Roy, *Soft Set Theory*, *Computers and Mathematics with Applications*, vol. 45, 2003, pp. 555–562.
- [8] F. Smarandache, Neutrosophic set, A generalisation of the intuitionistic fuzzy sets, *Inter. J. Pure Appl. Math.* 24 (2005) 287–297.
- [9] Aktas, H., Cagman, N.: Soft sets and soft groups. *Information Sciences* 177, 2726–2735 (2007).
- [10] Roy, A.R., Maji, P.K, A fuzzy soft set theoretic approach to decision making problems. *Journal of Computational and Applied Mathematics* 203, 412–418 (2007).
- [11] Yang, X., Yu, D., Yang, J., Wu, C.: Generalization of soft set theory: from crisp to fuzzy case. In: Cao, B.-Y. (ed.) *Fuzzy Information and Engineering: Proceedings of ICFIE-2007: Advances in Soft Computing*, vol. 40, pp. 345–355. Springer, Heidelberg (2007)
- [12] Majumdar, P., Samanta, S.K.: Similarity measure of soft sets. *New Mathematics and Natural Computation* 4(1), 1–12 (2008).
- [13] Xiao, Z., Gong, K., Zou, Y.: A combined forecasting approach based on fuzzy soft sets. *Journal of Computational and Applied Mathematics* 228, 326–333 (2009)
- [14] V. Torra and Y. Narukawa, On hesitant fuzzy sets and decision, in *Proceedings of the IEEE International Conference on Fuzzy Systems*, pp. 1378–1382, Jeju-do, Republic of Korea, August 2009.
- [15] Torra V, Hesitant fuzzy sets and decision, *International Journal of Intelligent Systems* 25 (6), 2010, pp. 395–407.
- [16] Xiao, Z., Gong, K., Zou, Y.: Hesitant fuzzy information aggregation in decision making. *Int.J.Approx.Reason.* 52(3), 395–407, 2011.
- [17] S.Das and S.Kar, The hesitant fuzzy soft sets and its application in decision-making, M.K.Chakraborty et al.(eds.), *Facts of Uncertainties and Applications*, Springer *Proceedings in Mathematics & Statistics* 125, 2015.
- [18] F. Smarandache, *Plithogeny, Plithogenic Set, Logic, Probability, and Statistics*. Pons Editions, Brussels, Belgium, 2017.
- [19] Shawkat Alkhazaleh, *Plithogenic Soft Set, Neutrosophic Sets and Systems*, University of New Mexico, Vol. 33, 2020.
- [20] J.Jayasudha, R.Nishanthi, Plithogenic Hesitant fuzzy Soft Set, *Advances in nonlinear variational inequalities*, International publications, 28(7s), 2025.