

A Fuzzy Erlang Loss Model for Capacity-Limited Electric Vehicle Charging Stations: α -Cut Blocking Analysis and Robust Charger Dimensioning

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Abstract

Some charging sites, such as a fixed set of bays at a depot, a parking facility or a roadside plaza, cannot hold a waiting line: a vehicle that finds every charger occupied leaves and is lost. The natural idealisation is the Erlang loss system, whose key quantity is the blocking probability. Its inputs, the arrival rate and the charging rate, are seldom known exactly. This paper treats them as triangular fuzzy numbers and studies the resulting fuzzy blocking probability, together with throughput, lost-vehicle rate and charger utilisation, through α -cuts. Using an inverse representation of the Erlang-B formula, we prove that blocking is strictly increasing in the offered load and strictly decreasing in the number of chargers, and that the exact α -cut of the fuzzy blocking probability is obtained by evaluating the formula at the corners (λ_L, μ_U) and (λ_U, μ_L) of the parameter box, whereas throughput requires the opposite corner pairing. We then define a robust charger count $c^*(\epsilon, \alpha)$ that meets a blocking target for every parameter value at level α , and prove that it exists, is monotone in ϵ and in α , and satisfies explicit lower and upper bounds. In an illustrative station (hypothetical data, not field measurements) the most-likely and worst-case dimensioning decisions differ by one or two chargers, a difference invisible to a crisp analysis. At full membership every result reduces to the classical Erlang-B model.

Keywords: Fuzzy queueing; Erlang loss system; electric vehicle charging; blocking probability; α -cut; capacity planning; robust dimensioning

2020 Mathematics Subject Classification: 60K25; 90B22; 03E72; 90B80

Rationale. 60K25 (queueing theory) covers the Erlang loss system. 90B22 (queues and service systems) covers the capacity-planning application. 03E72 (theory of fuzzy sets and fuzzy relations) covers the α -cut treatment of the uncertain rates. 90B80 (discrete location and assignment) is closest to the integer-valued charger-count decision studied in Section 4.4.

1 Introduction

1.1 Capacity-limited charging sites

A charging facility with a fixed number of bays and no waiting space behaves differently from the delay systems usually analysed by queueing models. A driver who finds all bays busy does not wait; the driver drives on. The performance question is then not the length of a queue but the fraction of arrivals turned away, and the planning question is how many chargers keep that fraction below an acceptable level. Stylised queueing models with blocking have been used for charging in parking facilities [1], and queueing formulations have been used more generally for charging-station planning and control [15, 2, 13]. Delay models of the M/M/1 and M/M/c type are treated in the classical texts [9, 7].

1.2 The Erlang loss model

With Poisson arrivals at rate λ , independent charging durations of mean $1/\mu$, and c chargers operating in parallel, the number of occupied chargers is a birth–death process on $\{0, \dots, c\}$. Its stationary distribution is a Poisson law truncated at c , and the probability that an arriving vehicle is blocked is the Erlang-B formula [6, 9, 7]

$$B(c, a) = (a^c/c!) / (\sum_{k=0}^c a^k/k!), \quad a = \lambda/\mu. \quad (1)$$

The quantity a is the offered load, measured in erlangs. A known feature of the formula is that it remains valid when charging durations follow any distribution with mean $1/\mu$, a property called insensitivity [8]; we use it only to indicate the scope of the formula and do not re-prove it.

1.3 Imprecise rates and fuzzy treatment

Neither λ nor μ is a fixed constant at a real site. The arrival rate moves with the time of day, the traffic around the site and the local vehicle population, and the charging rate moves with battery state, charger power and the vehicles present. The uncertainty is in the parameters and is distinct from the randomness of individual arrival and service times, which the Poisson and exponential structure already describes. Following Zadeh [14], we describe each imprecise rate by a fuzzy number and propagate it through the model with α -cuts [11, 12, 4].

1.4 Related work and the gap addressed

Fuzzy analyses of queues with delay are well developed, including multi-server systems solved by parametric nonlinear programming [3] and finite-capacity queues [5]. Within the literature that the authors consulted for this article, we did not locate a treatment that combines (i) a fuzzy Erlang-B loss model with exact monotonicity-based α -cuts, (ii) the decision-oriented quantity “charger count that meets a blocking target for every parameter value at level α ” and (iii) an application to EV charging. This is a statement about the sources consulted and not a claim that no such work exists.

1.5 Contributions

1. A proof, based on an inverse representation of (1), that $B(c, a)$ is strictly increasing in a and strictly decreasing in c (Theorem 4.1).
2. The exact α -cut of the fuzzy blocking probability, and a proof that throughput requires a different corner pairing from blocking and lost-vehicle rate (Theorem 4.2, Proposition 4.1).
3. A robust dimensioning rule $c^*(\epsilon, \alpha)$ with proved existence, monotonicity and two-sided bounds (Theorem 4.3).
4. A numerical study, with hypothetical data, showing that the modal and worst-case decisions differ.

2 Preliminaries

Definition 2.1 (Triangular fuzzy number). A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ with $a_1 < a_2 < a_3$ has membership function $\mu_{\tilde{A}}(x) = (x - a_1)/(a_2 - a_1)$ on $[a_1, a_2]$, $(a_3 - x)/(a_3 - a_2)$ on $[a_2, a_3]$, and 0 elsewhere.

Definition 2.2 (α -cut). For $0 \leq \alpha \leq 1$, $[\tilde{A}]^\alpha = \{x : \mu_{\tilde{A}}(x) \geq \alpha\} = [A_L(\alpha), A_U(\alpha)]$, where $A_L(\alpha) = a_1 + \alpha(a_2 - a_1)$ and $A_U(\alpha) = a_3 - \alpha(a_3 - a_2)$. The cuts are nested: $\alpha \leq \beta$ implies $[\tilde{A}]^\beta \subseteq [\tilde{A}]^\alpha$, and $[\tilde{A}]^1 = \{a_2\}$.

Definition 2.3 (Extension principle, α -cut form). Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be continuous and let $\tilde{X}, \tilde{Y}$ be fuzzy numbers with compact α -cuts. The fuzzy number $f(\tilde{X}, \tilde{Y})$ is the one whose α -cuts are $[f(\tilde{X}, \tilde{Y})]^\alpha = f([\tilde{X}]^\alpha \times [\tilde{Y}]^\alpha) = \{f(x, y) : x \in [\tilde{X}]^\alpha, y \in [\tilde{Y}]^\alpha\}$.

This is the standard consequence of Zadeh’s extension principle for continuous f and is the basis of the monotonicity technique of Buckley and Qu [4]; see also [10]. Because the box $[\tilde{X}]^\alpha \times [\tilde{Y}]^\alpha$ is compact and connected and f is continuous, the image is a compact interval. If f is monotone in each argument, the endpoints of this interval are attained at two opposite corners of the box.

Definition 2.4 (Erlang loss system). An Erlang loss system has Poisson arrivals at rate $\lambda > 0$, $c \geq 1$ identical parallel chargers, i.i.d. exponential charging times with rate $\mu > 0$, and no waiting room. An arrival that finds all c chargers busy is lost. The blocking probability is $B(c, a)$ in (1) with $a = \lambda/\mu$. Let N be the stationary number of busy chargers. We use $C(c, a) = E[N]$ for the carried load.

By the PASTA property, the long-run fraction of arrivals that are blocked equals $\Pr(N = c) = B(c, a)$, so the carried load is $C(c, a) = a(1 - B(c, a))$, the throughput (accepted-vehicle rate) is $\lambda(1 - B) = \mu C$, and the lost-vehicle rate is λB .

3 The Fuzzy Model

Assumptions. (A1) Arrivals are Poisson and charging durations are exponential in the underlying stochastic model. (A2) The c chargers are identical and no waiting is allowed. (A3) The arrival rate $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$ and the per-charger charging rate $\tilde{\mu} = (\mu_1, \mu_2, \mu_3)$ are triangular fuzzy numbers with $\lambda_1 > 0$, $\mu_1 > 0$, taken to be non-interacting. (A4) The number of chargers c is a crisp integer decision variable. Only λ and μ are fuzzy, because only they are imprecisely known at the planning stage; c is chosen by the planner.

Definition 3.1 (Fuzzy offered load and fuzzy blocking probability). The fuzzy offered load is $\tilde{a} = \tilde{\lambda}/\tilde{\mu}$ and the fuzzy blocking probability is $\tilde{B}_c = B(c, \tilde{\lambda}/\tilde{\mu})$, both defined through Definition 2.3. Write $\lambda_L, \lambda_U, \mu_L, \mu_U$ for the endpoints of the α -cuts of $\tilde{\lambda}, \tilde{\mu}$.

Since $\mu_L(\alpha) \geq \mu_1 > 0$, the quotient is always defined. The cut of $\tilde{a}$ is $[a_L(\alpha), a_U(\alpha)] = [\lambda_L/\mu_U, \lambda_U/\mu_L]$, because λ/μ is increasing in λ and decreasing in μ on the positive quadrant. Note that, unlike delay models, no stability condition $\rho < 1$ is needed: a loss system with finite capacity is stable for every $a > 0$.

4 Main Results

4.1 Monotonicity of the Erlang-B formula

Lemma 4.1. For integers $c \geq 0$ and $a > 0$,

$1/B(c, a) = \sum_{j=0}^c c^j a^{-j}$, $c^j = c(c-1)\cdots(c-j+1)$, $c^0 = 1$.

Proof. Divide numerator and denominator in (1) by $a^c/c!$. The denominator becomes $\sum_{k=0}^c (c!/k!) a^{k-c}$. Substituting $j = c - k$ gives $(c!/(c-j)!) a^{-j} = c^j a^{-j}$.

Theorem 4.1 (Monotonicity). For fixed $c \geq 1$, $a \mapsto B(c, a)$ is strictly increasing and differentiable with $\partial_a B > 0$. For fixed $a > 0$, $c \mapsto B(c, a)$ is strictly decreasing on $\{0, 1, 2, \dots\}$ with $B(0, a) = 1$ and $B(c, a) \rightarrow 0$ as $c \rightarrow \infty$.

Proof. Fix $c \geq 1$. By Lemma 4.1, $1/B = 1 + \sum_{j=1}^c c^j a^{-j}$ and every coefficient c^j with $1 \leq j \leq c$ is positive, so $\partial_a(1/B) = -\sum_{j=1}^c j c^j a^{-j-1} < 0$. Hence $1/B$ is strictly decreasing and B strictly increasing, with $\partial_a B = -B^2 \partial_a(1/B) > 0$.

Fix a . Compare $1/B(c+1, a)$ and $1/B(c, a)$ term by term. For $1 \leq j \leq c$, $(c+1)^j > c^j$ since each factor increases by one, and the case $j = c+1$ adds the positive term $(c+1)! a^{-(c+1)}$ that is absent for c . Thus $1/B(c+1, a) > 1/B(c, a)$, i.e. $B(c+1, a) < B(c, a)$. Finally $B(c, a) \leq a^c/c!$ because the denominator in (1) is at least 1, and $a^c/c! \rightarrow 0$.

4.2 α -cuts of the fuzzy blocking probability

Theorem 4.2 (α -cut of $\tilde{B}_c$). For every $c \geq 1$ and $\alpha \in [0, 1]$,

$$[\tilde{B}_c]^\alpha = [B(c, \lambda_L(\alpha)/\mu_U(\alpha)), B(c, \lambda_U(\alpha)/\mu_L(\alpha))]. \quad (2)$$

The cuts are nested, and $[\tilde{B}_c]^1 = \{B(c, \lambda_2/\mu_2)\}$. Moreover $\tilde{B}_c$ is a fuzzy number, and for fixed α both endpoints are strictly decreasing in c .

Proof. The map $(\lambda, \mu) \mapsto B(c, \lambda/\mu)$ is continuous on the positive quadrant. By Theorem 4.1 and since λ/μ is increasing in λ and decreasing in μ , it is strictly increasing in λ and strictly decreasing in μ . Therefore on the box $[\lambda_L, \lambda_U] \times [\mu_L, \mu_U]$ its minimum is attained at (λ_L, μ_U) and its maximum at (λ_U, μ_L) . By Definition 2.3 and connectedness of the box, the image is the interval between these values, which is (2). Nestedness follows from nestedness of the boxes. At $\alpha = 1$ the box is the single point (λ_2, μ_2) . A family of nested compact intervals, with the singleton at $\alpha = 1$, defines a normal convex fuzzy number. The last statement is Theorem 4.1 applied to the two endpoint loads.

Remark 4.1. Choosing the corners “lower with lower, upper with upper” would be wrong here. At $\alpha = 0$ and $c = 4$ in the example of Section 5, $B(4, \lambda_1/\mu_1) = 0.1687$ and $B(4, \lambda_3/\mu_3) = 0.2360$, whereas the correct interval is $[0.1146, 0.3107]$. The naive pairing understates the width of the uncertainty and understates the worst-case blocking.

4.3 Throughput, lost-vehicle rate and utilisation

Proposition 4.1. Let $T(\lambda, \mu) = \lambda(1 - B(c, \lambda/\mu)) = \mu C(c, \lambda/\mu)$, $R(\lambda, \mu) = \lambda B(c, \lambda/\mu)$ and $u(\lambda, \mu) = C(c, \lambda/\mu)/c$.

1. T is strictly increasing in λ and in μ . Hence $[\tilde{T}]^\alpha = [T(\lambda_L, \mu_L), T(\lambda_U, \mu_U)]$.
 2. R is strictly increasing in λ and strictly decreasing in μ . Hence $[\tilde{R}]^\alpha = [R(\lambda_L, \mu_U), R(\lambda_U, \mu_L)]$.
 3. u depends on (λ, μ) only through a and is strictly increasing in a . Hence $[\tilde{u}]^\alpha = [C(c, a_L)/c, C(c, a_U)/c]$.
- Proof. Let $C(a) = C(c, a) = a(1 - B(c, a))$. Under the stationary law $p_k \propto a^k/k!$ on $\{0, \dots, c\}$, which is an exponential family in $\log a$, one has $C(a) = E[N]$ and $C'(a) = \text{Var}(N)/a > 0$, since $c \geq 1$ makes N non-degenerate. Part (3) follows. For (1), $\partial_\lambda T = C'(a) > 0$. For μ , $\partial_\mu T = C(a) - aC'(a)$. From $C(a) = a(1 - B)$ we get $C'(a) = 1 - B - a \partial_a B$, hence $\partial_\mu T = a^2 \partial_a B > 0$ by Theorem 4.1. For (2), R is a product of the increasing positive functions λ and $B(c, \lambda/\mu)$ in λ , and decreases in μ because B does. The cut formulas follow as in the proof of Theorem 4.2. Thus blocking and lost-vehicle rate share the corner pairing $(\lambda_L, \mu_U) - (\lambda_U, \mu_L)$, while throughput, which grows with both rates, uses $(\lambda_L, \mu_L) - (\lambda_U, \mu_U)$. Throughput is bounded above by λ and also by $c\mu$.

4.4 Robust charger dimensioning

Definition 4.1 (Robust charger count). For a blocking target $\varepsilon \in (0, 1)$ and a level $\alpha \in [0, 1]$, set

$$c^*(\varepsilon, \alpha) = \min\{c \in \mathbb{N} : B(c, a_U(\alpha)) \leq \varepsilon\}, \quad a_U(\alpha) = \lambda_U(\alpha)/\mu_L(\alpha).$$

By Theorem 4.2, $B(c, a_U(\alpha))$ is the upper endpoint of $[\tilde{B}_c]^\alpha$, so $c^*(\varepsilon, \alpha)$ is the smallest number of chargers for which blocking is at most ε for every pair (λ, μ) with $\mu\lambda(\lambda) \geq \alpha$ and $\mu\mu(\mu) \geq \alpha$. The choice $\alpha = 1$ gives the modal (most likely) decision and $\alpha = 0$ the worst case over the support.

Theorem 4.3 (Properties of c^*). Fix $\varepsilon \in (0, 1)$ and $\alpha \in [0, 1]$, and write $a_U = a_U(\alpha)$.

1. (Existence and bounds) c^* is a well-defined positive integer and $\lceil a_U(1 - \varepsilon) \rceil \leq c^*(\varepsilon, \alpha) \leq \min\{c : a_U^c/c! \leq \varepsilon\}$.
2. (Sufficiency) Every $c \geq c^*(\varepsilon, \alpha)$ guarantees $B(c, \lambda/\mu) \leq \varepsilon$ for all (λ, μ) in the α -cut box, and $c^* - 1$ does not, since the corner (λ_U, μ_L) violates the target.
3. (Monotonicity) c^* is non-increasing in ε (a stricter target needs at least as many chargers) and non-increasing in α (a more demanding robustness level needs at least as many chargers).
4. (Crisp limit) $c^*(\varepsilon, 1)$ is the classical smallest c with $B(c, \lambda_2/\mu_2) \leq \varepsilon$.

Proof. (1) By Theorem 4.1, $B(c, a_U) \downarrow 0$, so the set is non-empty and has a least element. Upper bound: $B(c, a_U) \leq a_U^c/c!$, so any c with $a_U^c/c! \leq \varepsilon$ is feasible. Lower bound: since $C(c, a) = E[N] \leq c$ and $C = a(1 - B)$, we get $B \geq 1 - c/a$; the requirement $B \leq \varepsilon$ therefore forces $1 - c/a_U \leq \varepsilon$, i.e. $c \geq a_U(1 - \varepsilon)$. (2) For $c \geq c^*$, monotonicity in c gives $B(c, a_U) \leq \varepsilon$ and monotonicity in a gives $B(c, \lambda/\mu) \leq B(c, a_U)$ on the box. By minimality, $B(c^* - 1, a_U) > \varepsilon$. (3) If $\varepsilon' \leq \varepsilon$, any c feasible for ε' is feasible for ε . If $\alpha \leq \beta$ then $a_U(\beta) \leq a_U(\alpha)$ by nestedness, so $B(c, a_U(\beta)) \leq B(c, a_U(\alpha))$ and any c feasible at α is feasible at β . (4) At $\alpha = 1$, $a_U = \lambda_2/\mu_2$.

Corollary 4.1 (Price of robustness). The integer $\Delta(\epsilon) = c^*(\epsilon, 0) - c^*(\epsilon, 1)$ is non-negative; it equals the number of extra chargers needed to guard against the whole support rather than the modal value.

Remark 4.2 (Width of the blocking interval). Although both endpoints of $[\tilde{B}_c]^\alpha$ decrease in c , the width need not: it is bounded by the upper endpoint, which tends to zero, but it can rise first. For the example below the $\alpha = 0$ width is 0.114, 0.187, 0.213, 0.196 for $c = 1, \dots, 4$. No monotone-width claim is therefore made.

5 Numerical Illustration

All values below are hypothetical and chosen for illustration; they are not field measurements. Consider a site with $\tilde{\lambda} = (2.4, 3.0, 3.6)$ vehicles per hour and $\tilde{\mu} = (0.9, 1.0, 1.1)$ vehicles per hour per charger, i.e. a mean charging time of about one hour. Table 1 summarises the quantities and Table 2 gives the cuts. At $\alpha = 1$ the offered load is $a = 3$ erlangs and, for instance, $B(2, 3) = 4.5/8.5 = 0.5294$. All numbers were computed by the recursion $B(k, a) = aB(k-1, a)/(k + aB(k-1, a))$, $B(0, a) = 1$, and agree with direct evaluation of (1).

Table 1: Model quantities (illustrative data).

Quantity	Symbol	Value / definition	Unit
Arrival rate	$\tilde{\lambda}$	(2.4, 3.0, 3.6)	vehicles h ⁻¹
Charging rate per charger	$\tilde{\mu}$	(0.9, 1.0, 1.1)	vehicles h ⁻¹
Offered load	$\tilde{a}$	$\tilde{\lambda}/\tilde{\mu}$	erlang
Chargers	c	decision variable	count
Blocking target	ϵ	0.10, 0.05, 0.01	probability

Table 2: α -cuts of the fuzzy rates and offered load.

α	$[\tilde{\lambda}]^\alpha$	$[\tilde{\mu}]^\alpha$	$[\tilde{a}]^\alpha$
0.00	[2.400, 3.600]	[0.900, 1.100]	[2.1818, 4.0000]
0.25	[2.550, 3.450]	[0.925, 1.075]	[2.3721, 3.7297]
0.50	[2.700, 3.300]	[0.950, 1.050]	[2.5714, 3.4737]
0.75	[2.850, 3.150]	[0.975, 1.025]	[2.7805, 3.2308]
1.00	[3.000, 3.000]	[1.000, 1.000]	[3.0000, 3.0000]

Figure 2 shows the membership functions and Figure 1 the system. Table 3 gives the blocking cuts from (2); at $\alpha = 1$ they collapse to the crisp Erlang-B values 0.5294, 0.3462, 0.2061, 0.1101, 0.0522, 0.0219, 0.0081 for $c = 2, \dots, 8$, which verifies the crisp limit. Figure 3 displays the bands, and Figure 4 the shape of $\tilde{B}_c$ for three charger counts.

Table 3: Blocking-probability cuts $[\tilde{B}_c]^\alpha$.

c	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
2	[0.4279, 0.6154]	[0.4807, 0.5742]	[0.5294, 0.5294]
3	[0.2374, 0.4507]	[0.2918, 0.3994]	[0.3462, 0.3462]
4	[0.1146, 0.3107]	[0.1580, 0.2575]	[0.2061, 0.2061]
5	[0.0476, 0.1991]	[0.0751, 0.1518]	[0.1101, 0.1101]
6	[0.0170, 0.1172]	[0.0312, 0.0808]	[0.0522, 0.0522]
7	[0.0053, 0.0627]	[0.0113, 0.0385]	[0.0219, 0.0219]
8	[0.0014, 0.0304]	[0.0036, 0.0165]	[0.0081, 0.0081]

Table 4 reports the measures of Proposition 4.1. The throughput interval at $\alpha = 0$ lies below $\lambda_3 = 3.6$ and its upper end approaches λ_3 as c grows, while per-charger utilisation falls as chargers are added, as expected since

the same carried load is spread over more chargers. For $c = 4$, for example, the throughput cut is [1.9951, 2.7502], obtained at $(\lambda_L, \mu_L) = (2.4, 0.9)$ and $(\lambda_U, \mu_U) = (3.6, 1.1)$.

Table 4: Throughput T (vehicles h^{-1}), lost-vehicle rate R (vehicles h^{-1}) and utilisation u per charger; $[\cdot]_0$ is the $\alpha = 0$ interval and the superscript (1) marks the crisp value at $\alpha = 1$.

c	$[T]_0$	$T^{(1)}$	$[R]_0$	$R^{(1)}$	$[u]_0$	$u^{(1)}$
2	[1.218, 1.598]	1.412	[1.027, 2.215]	1.588	[0.624, 0.769]	0.706
3	[1.669, 2.240]	1.962	[0.570, 1.623]	1.038	[0.555, 0.732]	0.654
4	[1.995, 2.750]	2.382	[0.275, 1.118]	0.618	[0.483, 0.689]	0.595
5	[2.202, 3.118]	2.670	[0.114, 0.717]	0.330	[0.416, 0.641]	0.534
6	[2.315, 3.355]	2.844	[0.041, 0.422]	0.156	[0.357, 0.589]	0.474
7	[2.368, 3.489]	2.934	[0.013, 0.226]	0.066	[0.310, 0.536]	0.419
8	[2.389, 3.555]	2.976	[0.003, 0.110]	0.024	[0.272, 0.485]	0.372

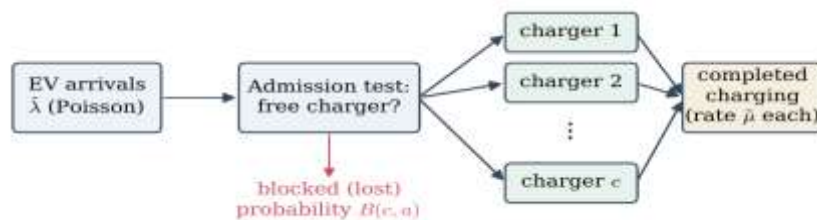


Figure 1: Capacity-limited charging station: Poisson arrivals at fuzzy rate $\tilde{\lambda}$, c parallel chargers each with fuzzy rate $\tilde{\mu}$, and no waiting room. An arrival finding all chargers busy is blocked with probability $B(c, a)$.

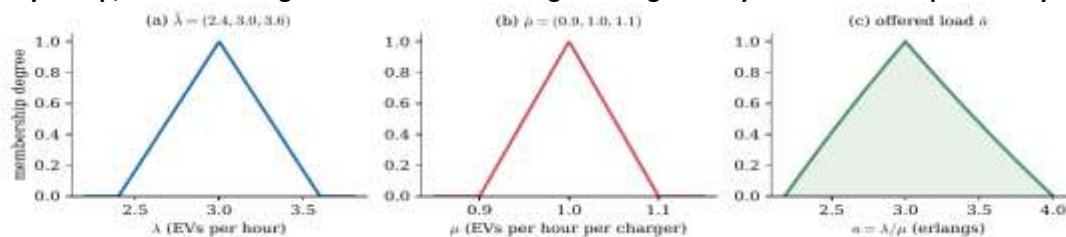


Figure 2: Membership functions of (a) the arrival rate, (b) the charging rate and (c) the offered load $\tilde{a}$, whose cuts are $[\lambda_L/\mu_U, \lambda_U/\mu_L]$ (illustrative data).

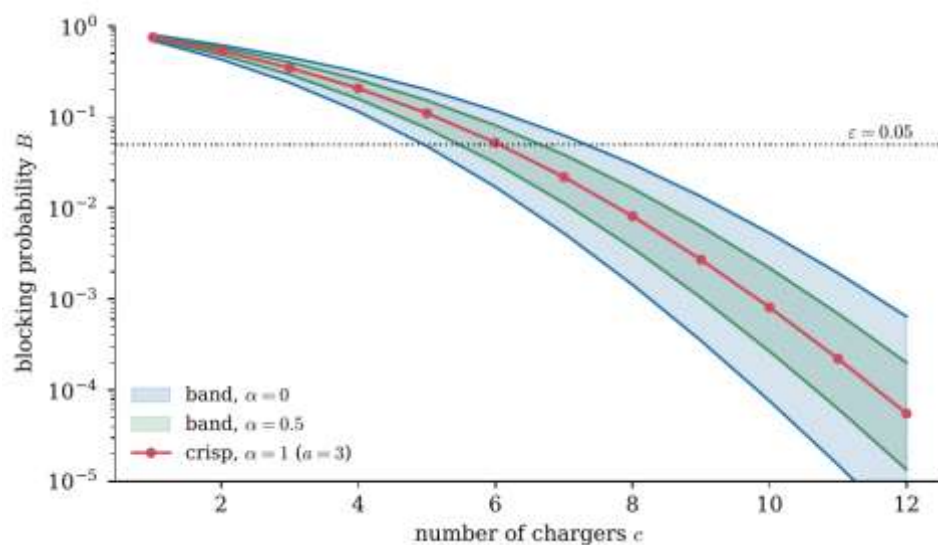


Figure 3: Blocking-probability bands $[\tilde{B}_c]^\alpha$ for $\alpha = 0$ and 0.5 against the number of chargers, with the crisp curve at $\alpha = 1$ ($a = 3$) and the target $\epsilon = 0.05$. Logarithmic vertical axis.

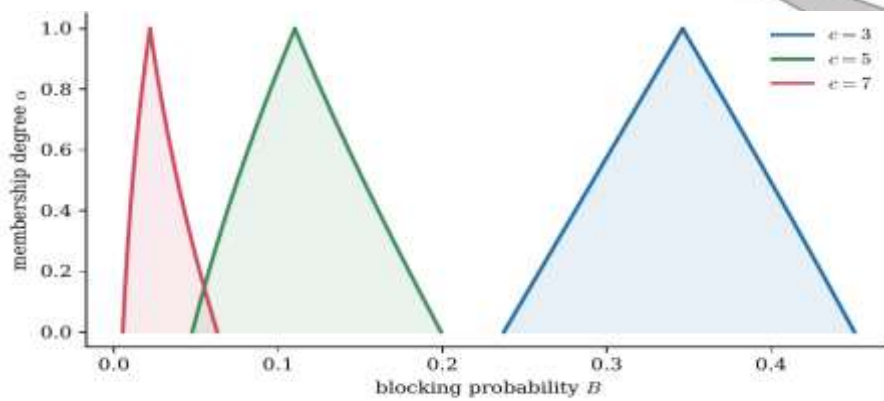


Figure 4: Membership functions of $\tilde{B}_c$ for $c = 3, 5, 7$, built from the α -cuts in (2). The functions are not triangular.

Robust dimensioning

Table 5 lists $c^*(\epsilon, \alpha)$ and Figure 5 plots it. For $\epsilon = 0.05$, a crisp analysis ($\alpha = 1$, $a_u = 3$) selects 7 chargers while the worst case over the support ($a_u = 4$) requires 8. For $\epsilon = 0.01$ the modal choice is 8 and the worst-case choice is 10, so $\Delta(0.01) = 2$. Table 6 verifies the two-sided bounds of Theorem 4.3 at $\alpha = 0$: the lower bound is weak (it only reflects carried load below capacity) and the upper bound is conservative, so the exact search over c is needed in practice.

Table 5: Robust charger count $c^*(\epsilon, \alpha)$.

ϵ	$\alpha = 0$	0.25	0.5	0.75	1
0.10	7	6	6	6	6
0.05	8	7	7	7	7
0.01	10	9	9	9	8

Table 6: Bounds of Theorem 4.3(1) at $\alpha = 0$ ($a_u = 4$).

ϵ	$\lceil a_u(1 - \epsilon) \rceil$	$c^*(\epsilon, 0)$	$\min\{c : a_u^c/c! \leq \epsilon\}$
0.10	4	7	12
0.05	4	8	12
0.01	4	10	14

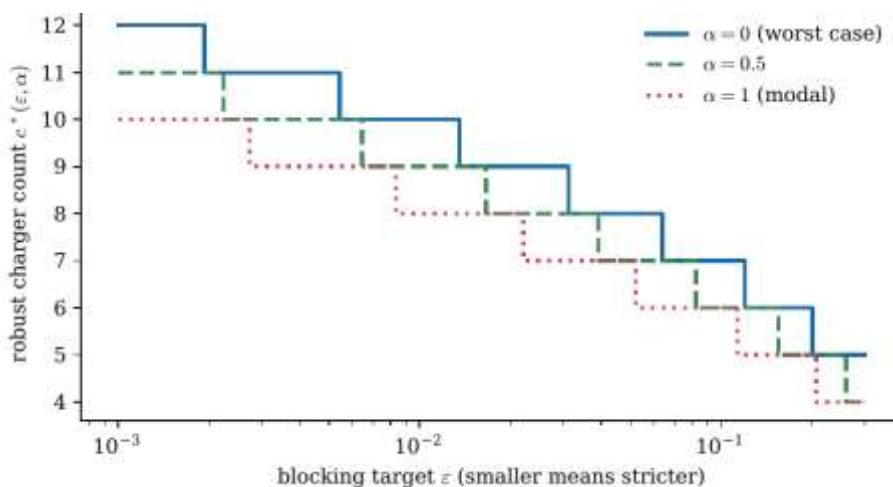


Figure 5: Robust charger count against the blocking target for $\alpha = 0, 0.5, 1$. The gap between the curves is the price of robustness.

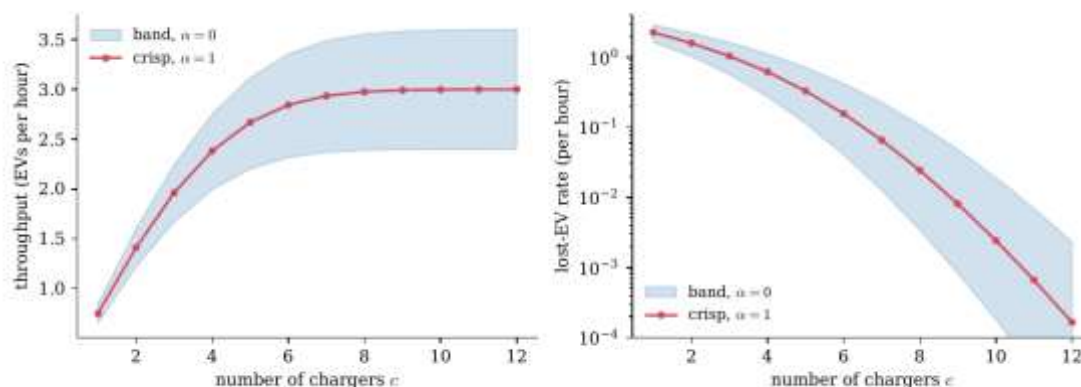


Figure 6: Throughput band (left) and lost-vehicle-rate band (right) at $\alpha = 0$ against the number of chargers, with the crisp values at $\alpha = 1$.

6 Discussion

The blocking interval is widest at moderate c (Remark above) and collapses at large c , because once capacity comfortably exceeds the offered load, parameter uncertainty matters little. In contrast, the uncertainty in the required capacity does not vanish: the gap between worst-case and modal c^* persists for stricter targets and grows from one to two chargers between $\varepsilon = 0.05$ and $\varepsilon = 0.01$ in the example. That is the mathematical content of the practical statement that guarding against tail demand matters most when the target is strict. The α -level acts as a robustness dial: smaller α widens the box and cannot lower c^* (Theorem 4.3(3)). The choice of α and of ε is a planning judgement; the model supplies the consequences of each choice and does not select them. Throughout, the quantitative statements concern the hypothetical data of Section 5; the theorems, in contrast, hold for any positive triangular parameters.

7 Limitations

The model assumes Poisson arrivals, a stationary arrival rate, identical chargers and no waiting. Charging power variation, battery state of charge, balking, reneging and time-varying demand are not modelled. The Erlang-B formula is insensitive to the charging-time distribution [8], which supports the loss probabilities, but the fuzzy extension treats λ and μ as non-interacting, so any dependence between arrival and charging behaviour is ignored. Triangular numbers are one convenient representation; Theorems 4.2 and 4.3 use only monotonicity and nestedness and so extend to trapezoidal numbers. No field data are used, so no claim of empirical validity is made. No cost model is introduced, so the answer is the smallest charger count that meets a service target, not a cost-minimising one.

8 Conclusion

We formulated a fuzzy Erlang loss model for charging sites without waiting space, proved strict monotonicity of the blocking probability in the offered load and in the number of chargers, and obtained exact α -cuts of blocking, throughput, lost-vehicle rate and utilisation, noting that throughput needs a different corner pairing from blocking. A robust charger count $c^*(\varepsilon, \alpha)$ was defined, shown to exist and to obey explicit bounds and monotonicity in ε and α , and shown to reduce to the classical dimensioning rule at full membership. In the illustrative example the worst-case and modal decisions differ by one to two chargers, which a crisp analysis cannot display. Natural extensions are finite waiting rooms, non-stationary arrivals, a cost-based objective, and calibration with measured site data.

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