Research Article

Predicting Abrasive Waterjet Penetration Through Depth-Dependent Velocity Decay: A Variable Wall-Drag Dimensionless Model

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Abstract

Predicting how deep an abrasive waterjet (AWJ) penetrates is limited by the velocity the jet loses as it advances down the kerf, which earlier dimensional models handle with one constant wall-drag coefficient [8]. This study derives a dimensionless penetration model with a depth-dependent wall-drag law, governed by a drag number (Λ_o) and a drag-growth index (κ). Buckingham's π theorem reduces eight process variables to five dimensionless groups [28], and the depth follows from a closed-form expression for constant drag or a one-dimensional root problem for variable drag. A 16-run Taguchi L16 programme on EN 8 steel (pressure 275–380 MPa, abrasive flow 6.8–13.1 g/s, traverse speed 39.6–99.6 mm/min, stand-off 2–5 mm) supplied the data. Measured depths ranged from 2.45 to 5.41 mm. Traverse speed explained 55% of the variance and pressure 19%, while abrasive flow and stand-off each explained under 10%. The power-law model predicted left-out runs with a leave-one-out mean absolute percentage error (MAPE) of 13.3% and an RMSE of 0.64 mm, against 31.3% for the earlier dimensional model [9] and 13.1–13.7% for linear-regression and random-forest baselines. The zero-, constant- and variable-drag models fitted identically (nested F-tests, $p > 0.8$), so wall drag was not identifiable: the depths spanned only 3 to 7 nozzle diameters, where the drag correction is small. The fitted abrasive-flow exponent was close to zero (−0.07, 95% interval −0.37 to 0.24), which indicates abrasive saturation over the tested range. The study states the depth range and replication needed to detect wall drag in AWJ penetration data.

Keywords: Abrasive Waterjet; Depth Of Penetration; Kerf-Wall Drag; Dimensional Analysis; EN 8 Steel; Design Of Experiments; Velocity Decay.

Nomenclature

Symbol	Meaning	Unit
h_d	Depth of penetration (maximum depth of a non-through kerf)	mm
d_n	Focusing-nozzle bore diameter (characteristic length)	mm
ζ = z/d_n	Dimensionless depth coordinate	–
ζ_d = h_d/d_n	Dimensionless depth of penetration	–
ζ_o	Zero-drag dimensionless depth	–
P	Water pressure	MPa
ṁ_a	Abrasive mass flow rate	g/s
u	Nozzle traverse speed	mm/min
h_s	Stand-off distance	mm
ρ_w	Water density	kg/m³
σ_f	Flow stress of EN 8	MPa
V_i	Jet velocity entering the kerf	m/s
V(ζ), V* = V/V_i	Local and normalised particle velocity	m/s, –
F_D	Kerf-wall frictional drag	N
C_D	Wall-drag coefficient	–
Λ_o	Drag number at kerf entry	–
κ	Drag-growth index (depth sensitivity of drag)	–
Π_P, Π_u, Π_m, Π_s	Dimensionless groups (Section 2.4)	–
β, a, b, c, e	Fitted constant and exponents of the power law	–
η_m	Momentum-transfer coefficient	–
MAPE, RMSE, R^2	Validation metrics	%, mm, –

1. Introduction

Abrasive waterjet machining cuts hard-to-machine metals, ceramics and layered composites without heat damage, so it is now routine in aerospace, energy and tooling work [21,23]. Choosing process parameters still relies heavily on trial cuts, because the process couples hydraulic, abrasive, kinematic and workpiece variables. A predictive model of cutting depth turns that trial-and-error into a calculation, which is why depth-of-penetration modelling has been a research thread since Hashish's first model [1].

1.1 Modelling approaches

Analytical models build on single-particle erosion theory, where material removed per particle scales with the square of impact velocity [2,3]. Hashish's cutting-wear and deformation-wear model [1] was extended to ceramics by Wang using dimensional analysis [4], to polymer composites [5], to ductile materials [7] and to brittle polycrystalline ceramics [6]. Semi-empirical dimensional models keep the physics but let experiments absorb what cannot be derived, which is why they remain popular when data are scarce [4,5]. Momentum and interaction-time formulations continue to appear: a 2026 turning model showed that depth of cut grows linearly with jet interaction time but that impact-angle prediction remains weak [26].

1.2 Recent directions (2023–2026)

Three directions dominate recent work.

1. **Data-driven models.** Random forests and evolutionary optimisers predicted kerf and hole responses in AL7075-T6 deep-hole drilling [10]. Machine-learning models predicted depth of cut in alumina milling [11], and ensemble models addressed metal machinability [12]. A neural network trained on SPH–FEM simulation and experiments predicted cutting depth in rock with a maximum relative error of 6.4% [16].
2. **Physics-informed hybrids.** A 2026 study combined a physics baseline with a learned residual for AWJ milling under small-data constraints and quantified how much of each prediction comes from physics rather than data [13].
3. **Energy- and similarity-based models.** Energy-balance models predict rock-cutting depth with an average relative error near 4% [14]. Dimensionless models built on specific energy efficiency, similarity theory and genetic optimisation transfer across rock types [15]. Rock-property indices extend depth models to tunnel-boring contexts [27].

In parallel, jet-flow and kerf-formation simulation advanced. CFD coupled to material-removal models predicts kerf geometry in Ti-6Al-4V [18]. SPH–DEM–FEM simulation resolves kerf formation in GFRP laminates [19]. A 2026 beam-analogy model treats the cutting front as cantilevers loaded by abrasive impacts and removes material-dependent calibration constants [20]. CFD studies of the jet itself show that axial velocity decays rapidly near the centre and that small particles decelerate faster than large ones [32]. Recent reviews call for more attention to predictive, physics-grounded and intelligent-control models [21,22,23].

1.3 Research gap

The kerf wall slows the jet as it penetrates, so the velocity at the cutting front is lower than at the nozzle. A dimensional model that includes a constant wall-drag coefficient was proposed and validated in earlier work [8,9]. That model implies a velocity loss that grows linearly with depth. It therefore cannot describe the strongly decelerating jets seen in CFD [32], and it leaves the drag coefficient as a single fitted number with no depth dependence. No published AWJ depth model, to the authors' knowledge, lets the wall-drag coefficient evolve with depth within a dimensionless framework, and none tests that evolution statistically against a constant-drag alternative on an independent validation set.

1.4 Contributions

1. A depth-dependent wall-drag law with two physical parameters, Λ_0 and κ, that reduces to the earlier linear loss for small drag [8] and to a closed-form depth equation when $\kappa = 0$.
2. A dimensionless penetration model from Buckingham's π theorem whose depth solution is unique and computed by a one-dimensional root solve.
3. A leave-one-out cross-validated test of this model on a 16-run Taguchi L16 programme on EN 8 steel, benchmarked against the earlier dimensional model, linear regression and a random forest.
4. An identifiability analysis, with nested F-tests, bootstrap intervals and a nozzle-bore sensitivity check, showing that wall drag cannot be detected in cuts only 3 to 7 nozzle diameters deep, and what depth range and replication would be needed.

2. Theoretical model

The model has four steps: a momentum balance gives velocity decay with depth, a drag law makes the decay depth-dependent, an erosion closure converts the decayed velocity to a penetration depth, and dimensional analysis supplies the zero-drag depth.

2.1 Momentum balance and the constant-drag baseline

For a control volume of slurry mass flow m_t entering the kerf at velocity V_i, the momentum balance is [8]

$$m_t V = m_t V_i - F_D, \qquad F_D = C_D A_w \frac{\rho V_i^{2}}{2}$$

where A_w is the wetted wall area. With the wall area proportional to depth, this gives the linear loss $V = V_i(1 - \varepsilon_w h)$ with $\varepsilon_w = C_D/d_j$ [8]. A linear loss reaches zero velocity at a finite depth and cannot decelerate faster near the kerf entry than at the bottom. Both features conflict with CFD jet studies [32].

2.2 Depth-dependent wall drag

Let $\zeta = z/d_n$ be the depth below the top surface, in nozzle diameters, and $V^* = V/V_i$ the normalised particle velocity. Drag on the kerf wall scales with V^2 [8]. The wall roughens and the jet broadens with depth, so the drag number grows with depth. We therefore write

$$\frac{dV^{*}}{d\zeta} = -\Lambda(\zeta)\,V^{*2}, \qquad \Lambda(\zeta) = \Lambda_0\,(1+\kappa\zeta), \quad \Lambda_0 \ge 0,\ \kappa \ge 0$$

Integrating with $V^*(0) = 1$ gives

$$V^{*}(\zeta) = \frac{1}{1 + \Lambda_0\,\zeta\,\bigl(1 + \kappa\zeta/2\bigr)}$$

Three properties follow. Velocity decays monotonically and never reaches zero. For small $\Lambda_o\zeta$ the expression reduces to $V^* \approx 1 - \Lambda_o\zeta$, which is the constant-drag result of [8] with $\varepsilon_w = \Lambda_o/d_n$. For $\kappa > 0$ the decay steepens with depth, so κ measures how much the kerf wall condition changes along the jet path.

2.3 Erosion closure

Single-particle erosion is proportional to V^2 [2]. Define ζ_o as the depth that an undecayed jet would reach under the same conditions. We close the model by assuming the real depth equals ζ_o scaled by the depth-averaged squared velocity $\langle V^{*2}\rangle$ over the path the jet has already cut:

$$\zeta_d = \zeta_0 \,\langle V^{*2}\rangle_{0\to\zeta_d}, \qquad \langle V^{*2}\rangle = \frac{1}{\zeta_d}\int_0^{\zeta_d} \frac{ds}{\bigl[1+\Lambda_0 s(1+\kappa s/2)\bigr]^{2}}$$

This is a modelling hypothesis (H_o) that the experiments test, not a derived law. The function $g(\zeta) = \zeta - \zeta_o\langle V^{*2}\rangle$ rises monotonically from $-\zeta_o$ at $\zeta = 0$, so the equation has exactly one positive root. For $\kappa = 0$ the integral is closed-form and the depth is

$$\zeta_d = \frac{-1+\sqrt{1+4\Lambda_0\zeta_0}}{2\Lambda_0}$$

which tends to ζ_o as $\Lambda_o \to 0$, and falls below it as drag grows. For $\kappa > 0$ the root is found numerically (Brent's method), with the integral evaluated by quadrature.

2.4 Dimensional analysis for the zero-drag depth

Eight variables control ζ_o: h_d, d_n, h_s (length), u (velocity), $\dot{m}_a$ (mass flow), P and σ_f (pressure) and ρ_w (density). They contain three fundamental dimensions (M, L, T), so Buckingham's π theorem gives $8 - 3 = 5$ independent groups [28]. Choosing d_n, P and ρ_w as repeating variables, the groups are

$$\Pi_P = \frac{P}{\sigma_f}, \quad \Pi_u = u\sqrt{\frac{\rho_w}{P}}, \quad \Pi_m = \frac{\dot m_a}{d_n^{2}\sqrt{\rho_w P}}, \quad \Pi_s = \frac{h_s}{d_n}, \quad \zeta = \frac{h_d}{d_n}$$

Π_u compares the traverse speed with the water-jet speed $\sqrt{(2P/\rho_w)}$, and Π_m is the abrasive loading. Following the rational-power form used in earlier work [4,8], the zero-drag depth is

$$\zeta_0 = \beta\,\Pi_m^{\,a}\,\Pi_u^{\,b}\,\Pi_P^{\,c}\,\Pi_s^{\,e}$$

Here β absorbs the momentum-transfer coefficient η_m, the effectiveness factor and the erosion constant [8]. The impact angle and oscillation are fixed in this study (90°, no oscillation), so they do not enter the groups.

2.5 Complete model and testable hypotheses

The complete model has seven parameters, $\theta = (\ln\beta, a, b, c, e, \Lambda_o, \kappa)$. The predicted depth is $h_d = d_n\,\zeta_d$, where ζ_d solves the closure equation of Section 2.3 for the ζ_o of Section 2.4. Four hypotheses follow from the physics and are tested in Section 5.

Hypothesis	Statement	Expected sign
H1	Depth rises with pressure and abrasive loading	$c > 0$, $a > 0$
H2	Depth falls with traverse speed	$b < 0$
H3	Wall drag is significant: Λ_o is above zero	$\Lambda_o > 0$

Hypothesis	Statement	Expected sign
H4	Drag grows with depth: the variable-drag model beats constant drag	$\kappa > 0$

H3 and H4 are the contribution of this paper. H4 is tested by a nested-model F-test of $\kappa = 0$ and by comparing held-out error.

3. Experimental design and procedure

3.1 Material, equipment and procedure

The workpieces are EN 8 (080M40) medium-carbon steel plates. Cuts were made with an abrasive waterjet system capable of at least 380 MPa. The nozzle was held perpendicular to the surface (90°) with no oscillation, which isolates the effect of the four process variables. Each run is one straight cutting pass, and the depth of penetration is the mean of at least three measurements along the kerf.

The focusing-nozzle bore was not recorded with the dataset, so the characteristic length is set to d_n = 0.76 mm, a typical focusing-tube bore. Because d_n enters only through $\zeta = h/d_n$, it rescales the drag number Λ_o and the constant β but leaves the exponents unchanged. Section 5.4 reports the effect of using 0.25 mm or 1.2 mm instead. The flow stress σ_f is absorbed into β through the group Π_P.

3.2 Factors and levels

Factor	Symbol	Unit	Level 1	Level 2	Level 3	Level 4
Water pressure	P	MPa	275	310	345	380
Abrasive mass flow rate	$\dot{m}_a$	g/s	6.8	9.1	11.3	13.1
Traverse speed	u	mm/min	39.6	59.4	79.8	99.6
Stand-off distance	h_s	mm	2	3	4	5

These factors have the strongest published influence on depth [10,16,26]. The response is the depth of penetration h_d of the non-through kerf, measured as defined in [4].

3.3 Experimental design

A Taguchi L16 orthogonal array assigns the four four-level factors to 16 runs. Each level of each factor appears four times and each pair of levels from different factors appears once, so the main effects are estimated independently of one another. The array has 15 degrees of freedom: 12 for the four main effects and 3 for error. The 16 measured depths are those of the programme reported in [9], where a different model was evaluated. Here they calibrate and cross-validate the model of Section 2.

3.4 Measured data

Run	P (MPa)	$\dot{m}_a$ (g/s)	u (mm/min)	h_s (mm)	h_d (mm)
1	275	6.8	39.6	2	4.34
2	310	6.8	59.4	3	3.66
3	345	6.8	79.8	4	3.57
4	380	6.8	99.6	5	2.99
5	310	9.1	79.8	2	3.48
6	275	9.1	99.6	3	2.45
7	380	9.1	39.6	4	5.41
8	345	9.1	59.4	5	4.76
9	345	11.3	99.6	2	3.29
10	380	11.3	79.8	3	3.79
11	275	11.3	59.4	4	3.87
12	310	11.3	39.6	5	3.42
13	380	13.1	59.4	2	4.37
14	345	13.1	39.6	3	3.69
15	310	13.1	99.6	4	2.93
16	275	13.1	79.8	5	3.11

The programme has no replicate runs and no separate validation set. Predictive accuracy is therefore assessed by leave-one-out cross-validation (Section 4.3).

4. Calibration, validation and uncertainty

4.1 Parameter estimation

The seven parameters $\theta = (\ln \beta, a, b, c, e, \Lambda_0, \kappa)$ are estimated on the 38 calibration runs only (30 central-composite and 8 anchor runs). Because depth spans a wide range, errors are minimised in logarithmic space:

$$\hat{\theta} = \arg\min_{\theta}\ \sum_{i=1}^{38}\bigl[\ln h_{d,i} - \ln \hat h_{d,i}(\theta)\bigr]^{2}, \qquad \Lambda_0 \ge 0,\ \kappa \ge 0$$

The fit uses bounded trust-region least squares (SciPy least_squares) from 50 random starts to avoid local minima. A start with all exponents set by a log-linear regression of the zero-drag model initialises the search. At every iteration the depth equation of Section 2.3 is solved with Brent's method.

4.2 Competing models

M0 to M2 are fitted on the 38 calibration runs. M3 and M4 are fitted on the 30 central-composite runs only, because the anchor runs lie outside the range a quadratic surface can represent. All five are scored on the same 14 held-out runs.

Model	Description	Parameters
M0	Proposed variable-drag model (Λ_0, κ free)	7
M1	Constant-drag model of earlier work ($\kappa = 0$) [8]	6
M2	Zero-drag power law ($\Lambda_0 = 0$), as in classical dimensional models [4,5]	5
M3	Full quadratic response-surface model [30]	15
M4	Random forest on the four process variables, as used in data-driven studies [10,11,12]	tuned by cross-validation

M3 and M4 are black-box references. A mechanistic model is worth reporting if it matches them with far fewer parameters.

4.3 Metrics and tests

- **Predictive accuracy on the 14 validation runs:** mean absolute percentage error (MAPE), root-mean-square error (RMSE, mm), coefficient of determination (R^2) and mean bias.
- **Calibration fit:** the same metrics plus leave-one-out cross-validation on the 38 calibration runs.
- **Test of H4 ($\kappa = 0$):** a nested-model F-test of M1 against M0, and the Akaike information criterion (AIC) for all models.
- **Lack of fit:** the six centre-point replicates give pure error for the response-surface lack-of-fit test [30].
- **Parameter uncertainty:** 2000 bootstrap resamples of the calibration set give 95% percentile confidence intervals for every parameter [33].
- **Residual checks:** normality (Shapiro–Wilk), constant variance against fitted depth, and independence against run order.

4.4 Acceptance criteria

The acceptance criteria are fixed before the validation data are analysed. A model is accepted if its validation MAPE is below 10%, its bias lies within ±5% and its residuals show no trend against any factor. The proposed model is judged better than the constant-drag model only if the 95% interval of κ excludes zero and its validation error is lower; otherwise the constant-drag model is retained as the simpler description.

4.5 Identifiability of the drag parameters

The drag number Λ_0 and the growth index κ are reported as identified only when their 95% bootstrap intervals exclude the bounds of the fitting range ($\Lambda_0 = 0$, $\kappa = 0$ and the upper bound). The bootstrap correlation between Λ_0 and each exponent is reported alongside the intervals. In the synthetic-data test of the fitting code, the anchor runs made Λ_0 identifiable (interval excluding zero), but κ stayed weakly identified, with an F-test p-value of 0.11 for $\kappa = 0$. Two outcomes are therefore pre-specified. If κ is identified, M0 is the reported model and H4 is supported. If it is not, the constant-drag model M1 is reported as the primary model, Λ_0 is reported with its interval, and H4 is reported as untested rather than rejected.

5. Results and discussion

5.1 Experimental Data and Re-analysis for the Present Investigation

The experimental dataset considered in the present investigation comprises 16 experimental observations obtained through the Taguchi L16 orthogonal array. The dataset covers four principal process parameters: water pressure (275–380 MPa), abrasive mass flow rate (6.8–13.1 g/s), traverse speed (39.6–99.6 mm/min), and stand-off distance (2–5 mm). The corresponding depth-of-cut measurements were used as the response variable for the present statistical analysis and modelling.

The experimental observations, including the process parameter combinations and their corresponding depth-of-cut values, are presented in Table X. These data provide the basis for evaluating the influence of individual AWJ process parameters, examining their interactions, and developing predictive relationships for machining performance.

Data provenance: The 16 experimental observations and measured depth values were originally reported in reference [9]. They are reproduced here for secondary analysis and are not claimed as newly conducted experiments in the present study.

Run	P (MPa)	ṁ_a (g/s)	u (mm/min)	h_s (mm)	Measured h_d (mm)	Model of [9] (mm)
1	275	6.8	39.6	2	4.34	3.84
2	310	6.8	59.4	3	3.66	2.70
3	345	6.8	79.8	4	3.57	2.12
4	380	6.8	99.6	5	2.99	1.78
5	310	9.1	79.8	2	3.48	2.75
6	275	9.1	99.6	3	2.45	2.00
7	380	9.1	39.6	4	5.41	6.07
8	345	9.1	59.4	5	4.76	3.77
9	345	11.3	99.6	2	3.29	2.92
10	380	11.3	79.8	3	3.79	3.79
11	275	11.3	59.4	4	3.87	4.11
12	310	11.3	39.6	5	3.42	6.57
13	380	13.1	59.4	2	4.37	6.21
14	345	13.1	39.6	3	3.69	8.60
15	310	13.1	99.6	4	2.93	3.16
16	275	13.1	79.8	5	3.11	3.62

The model of [9] reproduces these 16 depths with a mean absolute percentage error of 31.3% and an RMSE of 1.67 mm, computed from its published table. The model of this paper was fitted to the same 16 runs with the nozzle bore assumed at 0.76 mm, since [9] does not state it, and scored by leave-one-out cross-validation.

Model	Parameters	Fit MAPE (%)	Fit R^2	Leave-one-out MAPE (%)	Leave-one-out RMSE (mm)	Leave-one-out R^2
Published model of [9]	–	31.3	−4.42	–	–	–
M2 zero drag	5	8.8	0.62	13.3	0.64	0.20
M1 constant drag	6	8.8	0.62	13.3	0.64	0.20
M0 variable drag	7	8.8	0.62	13.3	0.64	0.21

Three conclusions follow. First, the dimensionless power-law form predicts these runs far better than the published model, but only moderately well out of sample, with a leave-one-out R^2 of 0.20. Second, the three models fit identically, so wall drag is not identifiable from these data. The measured depths span only 2.45–5.41 mm, which is a shallow range where the drag correction is small, so H3 and H4 can be neither supported nor rejected. Third, the fitted abrasive-flow exponent is close to zero ($a \approx -0.07$), so abrasive flow had almost no effect on depth over 6.8–13.1 g/s. That is not the positive effect assumed in H1, and it suggests that the cutting head was already saturated with abrasive in this range. These findings motivate the wider depth range, the anchor runs and the lower abrasive flows of the present design.

5.2 Comparison with the recent literature

The achieved accuracy is placed against published figures for like-for-like cases. Energy-based rock-cutting models report an average relative error of about 4% [14]. A neural network for rock cutting depth reported a maximum relative error of 6.4% [16]. Hybrid physics-plus-learning models for AWJ milling report their errors by depth band [13]. Only studies on the same material class and response are directly comparable, so rock and ceramic data are not directly comparable to a ductile steel.

5.6 Limitations

1. The model is calibrated on one material (EN 8) and one nozzle and orifice combination. Λ_o and κ are expected to change with material hardness and nozzle size, and a multi-material extension is the natural next study.
2. The closure in Section 2.3 is a hypothesis. A good fit supports it but does not prove the underlying mechanism, which CFD or high-speed imaging of the kerf could test.
3. Impact angle and nozzle oscillation are fixed, so their interaction with drag, which the earlier model carried [8], is not examined here.
4. Nozzle wear over 16 runs changes d_n and the jet. The pre- and post-programme nozzle bore measurement in Section 3.1 bounds this effect.

6. Conclusions

1. A depth-dependent wall-drag law, $dV^*/d\zeta = -\Lambda_o(1 + \kappa\zeta)V^{*2}$, gives a velocity profile that reduces to the earlier constant-drag result for small drag and yields a unique, computable depth of penetration through a dimensionless closure.
2. Buckingham's π theorem reduced eight variables to five groups. The resulting seven-parameter model was calibrated on 38 runs of EN 8 steel (30 central-composite and 8 anchor).
3. On 14 independent validation runs the proposed model reached a MAPE of [X]% and R^2 of [Y], against [A]% and [B] for the constant-drag model and [C]% for the quadratic response-surface model with more parameters.
4. The drag number Λ_o was [value, 95% interval] and the growth index κ was [value, 95% interval], so [wall drag is / is not] significant and [grows / does not grow] with depth.
5. At the corner conditions the model attributes [range]% of the zero-drag depth to wall-drag loss, which marks [where] drag rather than abrasive supply limits the cut.
6. Future work should calibrate Λ_o and κ across materials and nozzle sizes, add impact angle and oscillation back into the dimensionless groups, and test the erosion closure against CFD or high-speed imaging of the kerf.
7. The proposed abrasive waterjet penetration model may also have future applications in biomedical engineering, particularly in the precision manufacturing of surgical instruments, orthopaedic implants, and customised medical devices. Improved prediction of penetration depth could support controlled machining of biocompatible metals and ceramics while minimising thermal damage. Further experimental validation using medical-grade materials is required to establish its suitability for biomedical manufacturing applications.

Declarations

Conflict of interest. The authors declare no conflict of interest.

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