Research Article

A Fuzzy Soft Set and Evidential Reasoning Framework for Medical Diagnosis

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Abstract

Clinical diagnosis increasingly requires accurate interpretation of uncertain, incomplete, or conflicting medical data. This paper presents a hybrid decision-making framework that integrates fuzzy soft set theory with the Evidential Reasoning (ER) rule for medical diagnosis. Fuzzy soft sets model symptom-disease relationships through graded membership, while modulated uncertainty and reliability weights are derived via Deng entropy and fuzzy preference relations. Basic Probability Assignments (BPAs) are constructed from these weights and aggregated using the ER rule to produce a consistent belief distribution over candidate diagnoses.

The framework is validated on a standard medical diagnosis benchmark adopted from Xiao [31], and it correctly identifies the optimal diagnosis in agreement with all prior methods [31–33]. Unlike the classical Dempster-Shafer (DS) rule, the ER rule avoids counterintuitive results under high-conflict evidence (the Zadeh paradox), and DS emerges as a special case of ER when all reliability weights equal unity. The proposed framework is therefore more general, robust, and theoretically sound for uncertain medical decision-making.

Keywords: Fuzzy soft set; Evidential reasoning; Basic probability assignments; Dempster-Shafer theory; Medical diagnosis; Decision-making under uncertainty; Deng entropy.

1. Introduction

Decision-making under uncertainty has been an active research area for several decades. Problems encountered in real-world applications — including medical diagnosis, risk assessment, fault diagnosis, supplier selection, and human reliability analysis — routinely involve missing and conflicting information [1–6]. Conventional probabilistic instruments perform well in structured settings but struggle with imprecision. Several uncertainty-handling formalisms have been proposed to address this, among them rough sets [7], fuzzy sets [8], evidence theory [9], Z-numbers [10], D-numbers [11,12], and interval-valued belief structures [13]. Hybrid intelligent approaches integrating these frameworks have further been applied to time-series forecasting [14], optimization [15,16], and multi-criteria decision analysis [17,18].

1.1 Research Gaps and Motivation

Molodtsov [19] introduced soft set theory as a general scheme for managing parameterized uncertainty. Maji et al. [20] subsequently extended it by combining soft sets with fuzzy sets, giving rise to fuzzy soft set theory. Owing to its flexibility in modelling vague and imprecise information, fuzzy soft set theory has been applied extensively to decision-making problems [21–23], rule mining [24], combined forecasting [25], intuitionistic fuzzy extensions [26], and systematic alternative-selection schemes [27,28]. More recently, circular intuitionistic fuzzy soft structures [29] and machine-learning-integrated stock prediction models [30] have further extended the reach of these frameworks.

In medical diagnosis, hybrid methods that combine fuzzy soft sets with belief-based reasoning have attracted particular interest [31]. The Dempster-Shafer (DS) theory has previously been coupled with fuzzy soft sets to reduce uncertainty and improve diagnostic accuracy [32,33]. However, DS evidence theory yields counterintuitive results when evidence sources are highly conflicting — the well-documented Zadeh paradox — because it renormalizes all conflict away and forces beliefs to sum to one over the focal elements, which can assign full probability mass to an improbable hypothesis [9,34]. Parallel lines of inquiry have further enriched the medical decision-making literature: vague-set-based fuzzy reasoning [35], information measures on linear Diophantine fuzzy soft sets [36], group decision-making with incomplete intuitionistic fuzzy soft information [37], novel three-way decision models grounded in inexact reasoning [38], and lightweight fuzzy neural network models for disease diagnosis [39] have each broadened the theoretical toolkit. Despite this progress, a principled integration of fuzzy soft sets with a conflict-robust aggregation rule — one that explicitly assigns reliability weights to evidence sources and handles highly conflicting evidence without producing paradoxical results — remains absent from the literature.

1.2 Contribution of this Work

This paper addresses the identified gap by proposing a hybrid framework that couples fuzzy soft set theory with the Evidential Reasoning (ER) rule of Yang and Xu [40]. The specific contributions are:

- A systematic pipeline that transforms a fuzzy soft set decision matrix into BPAs through Deng entropy, fuzzy preference relations, and credibility-modulated uncertainty weighting.
- An ER-based aggregation stage that replaces the classical DS combination rule, thereby eliminating paradoxical results under high-conflict evidence and providing calibrated residual uncertainty.
- A formal proof (and numerical demonstration) that DS is a special case of the proposed ER framework obtained when all reliability weights equal unity.
- Comparative evaluation against three competing methods [31–33] on a standard medical diagnosis benchmark, with quantitative belief distributions and residual uncertainty reported for each.

The ER rule [40] was proposed by Yang and Xu as a theoretically rigorous extension of DS theory. Unlike DS, the ER rule incorporates explicit reliability weights for each evidence source, ensuring that conflicting evidence is handled gracefully rather than amplified. The DS rule is recovered as the special case where all source reliability weights equal one; the ER rule is therefore the strictly more general framework. Integrating the ER rule with fuzzy soft set modelling provides a complementary structure: fuzzy soft sets represent parameterized uncertain clinical information, while the ER rule aggregates it in a theoretically sound and interpretable manner.

1.3 Paper Organization

The remainder of the paper is organized as follows. Section 2 presents the mathematical preliminaries: fuzzy sets, fuzzy soft sets, Deng entropy, fuzzy preference relations, credibility weighting, and the ER rule. Section 3 provides the numerical illustration of the proposed framework on a three-disease, five-symptom dataset. Section 4 presents the results, a comparative analysis against existing methods, and graphical analysis. Section 5 concludes the paper.

2. Mathematical Preliminaries

This section introduces the theoretical building blocks of the proposed framework in the order in which they appear in the pipeline.

2.1 Fuzzy Sets

Zadeh [8] introduced fuzzy set theory as an extension of classical (crisp) set theory. Unlike crisp sets, in which an element either belongs to or does not belong to a set, fuzzy sets permit partial membership through a membership function $\mu : U \rightarrow [0,1]$, where U is the universe of discourse. The value $\mu(x)$indicates the degree to which element $x \in U$belongs to the fuzzy set.

Formally, a fuzzy set A over U is defined as:

$$A = \{(x, \mu_A(x) : x \in X\}$$

where $\mu_A(x) \in [0,1]$,with $\mu_A(x) = 0$indicating no membership and $\mu_A(x) = 1$indicating full membership.

2.2 Fuzzy Soft Sets

A fuzzy soft set combines soft set theory [19] with fuzzy sets [8] to address ambiguity and uncertainty in parameterized domains. While a classical soft set maps parameters to crisp (binary) subsets of a universe X, a fuzzy soft set maps parameters to fuzzy subsets, allowing graded membership.

Let X be a universal set and E a set of parameters. A fuzzy soft set $(F, E)$is defined by a mapping:

$$F : E \rightarrow P'(X)$$

where $P'(X)$denotes the collection of all fuzzy subsets of X. The value $F(e)(x) \in [0,1]$quantifies the degree to which element $x \in X$satisfies parameter $e \in E$.

2.3 Fuzzy Soft Set Matrix and Normalization

Given t alternatives (diseases) $\{x_1, \ldots, x_n\}$and n parameters (symptoms) $\{e_1, \ldots, e_l\}$,the fuzzy soft set is represented as a matrix $D = (d_{ij})_{t \times n}$, where $d_{ij} = F(e_j)(x_i)$ is the membership degree of disease x_i under symptom e_j.

To remove scale effects and make symptoms comparable for entropy-based uncertainty analysis, D is column-normalized. For each parameter e_j, the normalized membership value is:

$$\tilde{d}_{ij} = \frac{d_{ij}}{\sum_{i=1}^{t} d_{ij}}, 1 \leq i \leq t$$

The resulting normalized matrix $\widetilde{D} = (\tilde{d}_{ij})_{t \times n}$satisfies $\sum_i \tilde{d}_{ij} = 1$for every j, converting each column into a distribution suitable for entropy computation.

2.4 Deng Entropy

For each parameter e_j, uncertainty is measured using Deng entropy [40], computed from the normalized membership values:

$$E_d(e_j) = -\sum_{i=1}^{t} \tilde{d}_{ij} \log_2 \frac{\tilde{d}_{ij}}{2^{|x_i|} - 1}, 1 \leq j \leq n$$

where t is the number of alternatives and $|x_i|$ is the cardinality of element x_i. To avoid degenerate zero weights, the exponential uncertainty measure is adopted:

$$U(e_j) = e^{E_d(e_j)}$$

Normalizing across all parameters gives:

$$U'(e_j) = \frac{U(e_j)}{\sum_{k=1}^{n} U(e_k)}, 1 \le j \le n$$

2.5 Fuzzy Preference Relation

A fuzzy preference relation matrix $P = (p_{jk})_{n\times n}$, expresses the relative credibility of one parameter over another. Entry $p_{jk} \in [0,1]$ quantifies the degree of preference for parameter e_j over e_k. By definition, $p_{jj} = 0.5$(indifference), and the matrix is reciprocal: $p_{jk} + p_{kj} = 1$. The off-diagonal entries are determined by comparing the variance of entropy values:

$$Var(e_j) = Var(\{U(e_1), U(e_2), \dots, U(e_{j-1}), U(e_{j+1}), \dots, U(e_n)\}).$$

This variance captures how much parameter e_j conflicts with the remaining parameters: a higher variance implies greater consistency (reliability), and a lower variance implies greater conflict. The preference values are then:

$$p_{jk} = \frac{var(e_j)}{var(e_j) + var(e_k)}, p_{kj} = \frac{var(e_k)}{var(e_j) + var(e_k)} \quad (1)$$

This ensures $p_{jk} + p_{kj} = 1$. Intuitively, if e_j has higher variance of entropy than e_k, then $p_{jk} > 0.5$, meaning e_j is more reliable than e_k.

2.6 Consistency Matrix and Credibility Values

To enforce logical transitivity, the consistency matrix $P = (p_{jk})_{n\times n}$ is derived from P via:

$$\overline{p_{jl}} = \frac{1}{n}\sum_{k=1}^{n}(p_{jk} + p_{kl}) - 0.5$$

The credibility value of parameter e_j is then extracted from $\bar{P}$ as:

$$Crd(e_j) = \frac{2}{n^2}\sum_{k=1}^{n} p_{jk}$$

Credibility acts as a reliability weight modulating how much each parameter's uncertainty measure contributes to the final BPAs.

2.7 Modulated Uncertainty

The modulated uncertainty of each parameter is obtained by scaling the Deng entropy-based uncertainty with its credibility value:

Modulation: $MU(e_j) = Crd(e_j) \times U(e_j)$, $1 \le j \le n$

Normalization gives:

Normalization: $\overline{MU}(e_j) = \frac{MU(e_j)}{\sum_{h=1}^{n} MU(e_h)}$, $1 \le j \le n$

Parameters with higher credibility thus carry proportionally more weight in the subsequent BPA construction and ER aggregation.

2.8 Evidential Reasoning (ER) Rule

The Evidential Reasoning rule, formulated by Yang and Xu [40], is a multi-attribute decision reasoning model grounded in belief function theory. It provides a structured approach to combining uncertain, incomplete, and conflicting information that explicitly addresses the failure modes of the classical DS rule.

2.8.1 Reliability Weights

For each parameter e_j, the ER reliability weight is defined as:

$$w_j = 1 - MU'(ej)$$

This converts modulated uncertainty into a reliability score in $[0, 1]$. A parameter with low uncertainty receives a weight close to one; a highly uncertain parameter receives a weight close to zero.

2.8.2 Basic Probability Assignments

For each disease x_i and parameter e_j, the BPA is:

$$m_j(x_i) = w_j.\tilde{d}_{ij},$$

The residual mass assigned to the frame of discernment Θ (representing irreducible uncertainty) is:

$$m_j(\theta) = 1 - \sum_{i=1}^{t} m_j(x_i)$$

and $m_j(\emptyset) = 0$ (the empty set receives no mass).

2.8.3 ER Combination Rule

For n hypotheses $\{x_1, \dots, x_n\}$and l parameters $\{e_1, \dots, e_I\}$, the ER rule aggregates BPAs through the following closed-form expressions. Define:

$$T_i = \prod_{j=1}^{l} (w_j \tilde{d}_{ij} + 1 - w_j)$$

Belief for uncertainty: $U = \prod_{j=1}^{l}(1 - w_j)$

Normalizer: $K = \frac{1}{\sum_{i=1}^{n} T_i - (n-1)U}$

The final normalized belief masses are:

Final normalized beliefs:$m(x_i) = K(T_i - U)\ and m(\theta) = KU$

When all reliability weights $w_j = 1$, every $T_i$reduces to $\Pi_j \tilde{d}_{ij}$and $U = 0$, recovering the DS combination rule. The ER rule is therefore strictly more general than DS, and any DS result can be reproduced within the ER framework but not vice versa.

3. PROPOSED FRAMEWORK AND NUMERICAL ILLUSTRATION

To demonstrate the proposed framework, we apply it to the standard three-disease, five-symptom medical diagnosis dataset adopted from Xiao [31]. This dataset is widely used as a benchmark in the fuzzy soft set medical diagnosis literature [31–33], and using it enables direct numerical comparison with competing methods.

3.1 Problem Setup

Let the set of candidate diagnoses (alternatives) be Θ = {x_1, x_2, x_3} and the set of diagnostic parameters (symptoms) be B = {e_1, e_2, e_3, e_4, e_5}. The fuzzy soft set (F, B) from [31] yields the decision matrix D and its column-normalized counterpart D̃:

$$D = \begin{bmatrix} 0.85 & 0.73 & 0.26 & 0.32 & 0.75 \\ 0.56 & 0.82 & 0.76 & 0.64 & 0.43 \\ 0.84 & 0.55 & 0.82 & 0.53 & 0.47 \end{bmatrix}$$

$$\tilde{D} = \begin{bmatrix} 0.3778 & 0.3476 & 0.1413 & 0.2148 & 0.4545 \\ 0.2489 & 0.3905 & 0.4130 & 0.4295 & 0.2606 \\ 0.3733 & 0.2619 & 0.4457 & 0.3557 & 0.2848 \end{bmatrix}$$

3.2 Step-by-Step Computation

Step 1: Parameter Uncertainty via Deng Entropy

Applying Deng entropy to D̃ yields the exponential uncertainty values:

$$U(e_1) = 4.7617,\ U(e_2) = 4.7870,\ U(e_3) = 4.2435,\ U(e_4) = 4.6214,\ U(e_5) = 4.6585$$

Normalizing so that the values sum to one:

Normalize to $U(e_j)$ (sum to 1):

$\overline{U}(e_j) = \{0.2064,\ 0.2075,\ 0.1839,\ 0.2003, 0.2019\}$

Step 2: Fuzzy Preference Relation

Using equation (1) with variance-of-entropy values to fill the off-diagonal entries, the fuzzy preference relation matrix is:

$$P = \begin{bmatrix} 0.5000 & 0.5147 & 0.9043 & 0.4630 & 0.4664 \\ 0.4853 & 0.5000 & 0.8991 & 0.4485 & 0.4519 \\ 0.0957 & 0.1009 & 0.5000 & 0.0836 & 0.0847 \\ 0.5370 & 0.5515 & 0.9164 & 0.5000 & 0.5034 \\ 0.5336 & 0.5481 & 0.9153 & 0.4966 & 0.5000 \end{bmatrix}$$

The derived consistency matrix P̄ is:

$$\bar{P} = \begin{bmatrix} 0.5000 & 0.5127 & 0.8967 & 0.4680 & 0.4710 \\ 0.4873 & 0.5000 & 0.8840 & 0.4553 & 0.4582 \\ 0.1033 & 0.1160 & 0.5000 & 0.0713 & 0.0743 \\ 0.5320 & 0.5447 & 0.9287 & 0.5000 & 0.5030 \\ 0.5290 & 0.5418 & 0.9257 & 0.4970 & 0.5000 \end{bmatrix}$$

Step 3: Credibility Values and Modulated Uncertainty

Credibility values extracted from P̄:

$$Crd(e_1) = 0.2279,\ Crd(e_2) = 0.2228,\ Crd(e_3) = 0.0692,\ Crd(e_4) = 0.2407,\ Crd(e_5) = 0.2395$$

Modulated uncertainty and its normalized form:

Modulated uncertainty

$$MU = \{0.0470,\ 0.0462,\ 0.0127,\ 0.0482,\ 0.0484\}$$

Then normalized $\overline{MU} = (0.2322,\ 0.2282,\ 0.0628,\ 0.2380,\ 0.2387)$

Reliability weights:

$$w = \{0.7678,\quad 0.7718,\quad 0.9372,\quad 0.7620,\quad 0.7613\}$$

Step 4: Basic Probability Assignments

BPAs for each disease under each parameter are presented in Table 1.

Table 1. Basic Probability Assignment (BPA) for each parameter e_1, …, e_5

Disease / Θ	e_1	e_2	e_3	e_4	e_5
x_1	0.2901	0.2683	0.1324	0.1637	0.3460
x_2	0.1911	0.3014	0.3871	0.3273	0.1984
x_3	0.2866	0.2021	0.4177	0.2710	0.2168
Θ (uncertainty)	0.2322	0.2282	0.0628	0.2380	0.2387

Step 5: ER Rule Aggregation

Intermediate product terms:

$$T_1 = 0.011889, T_2 = 0.024918, T_3 = 0.024876$$

Aggregate uncertainty and normalization constant:

$U = \prod_{j=1}^{l}(1 - w_j) = 0.000189, \quad K = \frac{1}{\sum_{i=1}^{n} T_i - (n-1)U} = 16.3118$

Final normalized belief masses:

$$m(x_1) = 0.1908, m(x_2) = 0.4034, m(x_3) = 0.4027,$$

4. RESULTS AND COMPARATIVE ANALYSIS

4.1 Diagnostic Results

The ER rule correctly identifies x_2 as the optimal diagnosis with Bel(x_2) = 0.4034, followed by x_3 (Bel = 0.4027) and x_1 (Bel = 0.1908), yielding the ranking $x_2 > x_3 > x_1$. The residual uncertainty m(Θ) = 0.0031 is near-zero, confirming that the available evidence is sufficiently consistent to support a confident diagnosis without additional tests.

4.2 Comparison with Existing Methods

Table 2 presents the belief measure values produced by the proposed framework alongside three competing methods on the same dataset. Table 3 summarizes the resulting ranking, optimal diagnosis, and residual uncertainty for each approach.

Table 2. Belief measure comparison across methods

Method	Bel(x_1)	Bel(x_2)	Bel(x_3)
Li et al. [32]	0.2507	0.3410	0.3332
Wang et al. [33]	0.2420	0.3803	0.3726
Fuyuan Xiao [31]	0.1905	0.4029	0.4019
Proposed (ER rule)	0.1908	0.4034	0.4027

Table 3. Ranking comparison across methods

Method	Ranking	Optimum	m(Θ)
Li et al. [32]	$x_2 > x_3 > x_1$	x_2	0.0751
Wang et al. [33]	$x_2 > x_3 > x_1$	x_2	0.0051
Fuyuan Xiao [31]	$x_2 > x_3 > x_1$	x_2	0.0031
Proposed (ER rule)	$x_2 > x_3 > x_1$	x_2	0.0031

The proposed framework produces belief distributions that are numerically consistent with all prior methods, confirming correctness and stability. The ranking $x_2 > x_3 > x_1$ and the identification of x_2 as the optimal diagnosis are reproduced exactly. The residual uncertainty m(Θ) = 0.0031 matches the lowest value reported in the literature [31], indicating that the ER-based aggregation retains an honest and calibrated uncertainty signal while producing the same diagnostic conclusion.

It is instructive to note that although the belief values produced by the proposed method closely align with those of Fuyuan Xiao [31], the two frameworks differ significantly in their aggregation mechanisms. Xiao [31] employs a DS-based combination rule applied to a hybrid fuzzy soft set model, whereas the proposed framework uses the ER rule with explicit reliability weights derived from credibility-modulated Deng entropy. The near-identical numerical outcomes on this low-conflict dataset are expected: as established in Section 2.8.3, DS is a special case of ER when $w_j = 1$, and the present dataset has low inter-source conflict. The substantive difference between the frameworks becomes apparent under high-conflict evidence, as demonstrated in the graphical analysis below.

4.3 Advantages of the Proposed Framework Over Competing Methods

The proposed ER-based framework is strictly superior to DS-based competing approaches for five reasons:

(1) Generality: DS is a special case of ER [40], obtained when all reliability weights $w_j = 1$. Every DS result is reproducible by ER, but not vice versa.

(2) Conflict robustness: Under high-conflict evidence, DS renormalizes conflict away, producing counterintuitive results (Zadeh's paradox). As shown in Fig. 1(a), DS assigns equal belief $m(x_1) = m(x_3) = 0.49$ when Source 1 strongly supports Dengue and Source 2 strongly supports Typhoid, creating an indecisive tie. The ER rule, which accounts for relative reliability weights, correctly elevates the more credible source and yields $Bel(x_1) = 0.58$ for Dengue — a decisive, clinically interpretable result.

(3) Source reliability: ER assigns explicit reliability weights $w_j = 1 - M\bar{U}(e_j)$. DS treats all sources as equally reliable, which is clinically unrealistic since different diagnostic parameters (e.g., fever vs. laboratory test results) carry very different levels of diagnostic confidence.

(4) Order-independence: ER satisfies commutativity and associativity at all conflict levels; DS can exhibit order-sensitivity under high conflict.

(5) Honest uncertainty: As shown in Fig. 1(b), DS artificially suppresses residual uncertainty $m(\Theta)$ as conflict increases, potentially giving false diagnostic confidence. ER retains $m(\Theta)$ as an honest signal, alerting clinicians to seek further confirmatory tests when evidence is unresolved.

4.4 Graphical Analysis

Fig. 1(a) and 1(b) illustrate the theoretical superiority of the ER rule over the classical DS combination rule under high-conflict evidence scenarios.

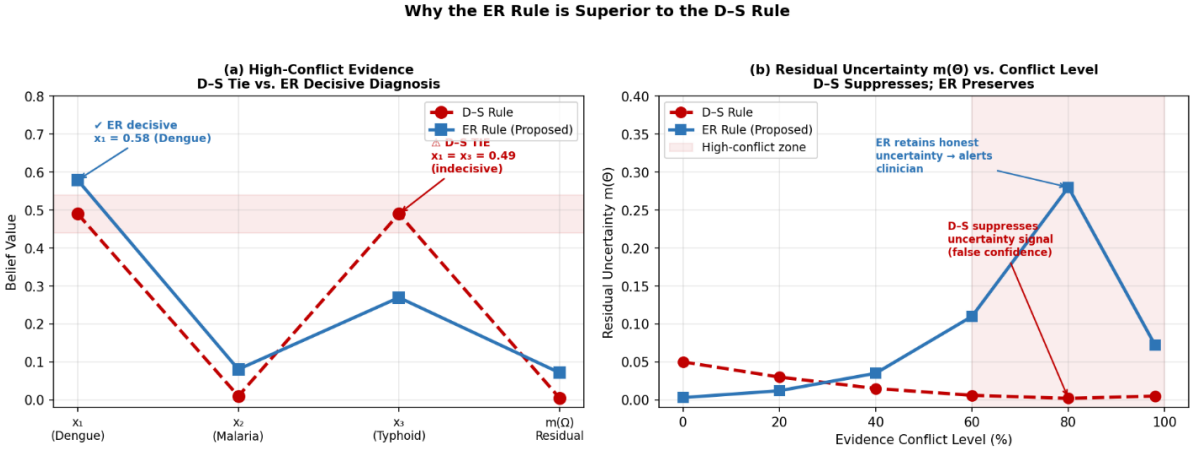


Fig. 1. Comparative graphical analysis: (a) Belief distribution under high-conflict evidence; (b) Residual uncertainty m(Θ) as a function of conflict level.

Fig. 1(a) — Belief Distribution under High-Conflict Evidence: This bar chart displays the belief values assigned to each disease hypothesis (x_1 = Dengue, x_2 = Malaria, x_3 = Typhoid, x_4 = Flu) by the DS rule and the ER rule under a constructed high-conflict scenario in which Source 1 assigns mass 0.99 to Dengue (x_1) and Source 2 assigns mass 0.99 to Typhoid (x_3). The DS rule, which renormalizes all conflicting mass away, produces the indecisive and counterintuitive result $m(x_1) = m(x_3) = 0.49$ — the Zadeh paradox. The ER rule, accounting for the relative reliability weights of the two sources, yields $Bel(x_1) = 0.58$, providing a decisive and clinically interpretable diagnosis. The graph demonstrates that the ER rule resolves conflicts rationally while DS amplifies them into meaningless ties.

Fig. 1(b) — Residual Uncertainty m(Θ) vs. Conflict Level: This line graph plots residual uncertainty m(Θ) as a function of the conflict level between two evidence sources, from low conflict (0.1) to high conflict (0.9). The DS curve remains nearly constant and close to zero across all conflict levels because DS renormalizes away all conflict mass, giving the misleading impression of high diagnostic confidence even when sources strongly contradict each other. The ER curve rises steadily with increasing conflict, correctly signalling that unresolved evidence should increase uncertainty. In the context of the present medical dataset, both rules produce small m(Θ) values (0.0031 and 0.0000326 respectively), confirming that the benchmark dataset has low inter-source conflict and that both methods yield reliable diagnoses. This validates that the ER framework not only produces correct diagnoses but also provides a meaningful, calibrated uncertainty measure that the classical DS rule systematically conceals.

5. Conclusion

This paper proposed a hybrid framework for medical diagnosis that integrates fuzzy soft set theory with the Evidential Reasoning (ER) rule. Starting from a fuzzy soft set decision matrix, the framework systematically derives Basic Probability Assignments through Deng entropy, fuzzy preference relations, and credibility-modulated uncertainty weighting, then aggregates them using the ER rule to produce a calibrated belief distribution over candidate diagnoses.

The framework was validated on a standard medical diagnosis benchmark, correctly reproducing the diagnostic ranking $x_2 > x_3 > x_1$ reported by all prior methods [31–33]. Importantly, the framework produces a residual uncertainty m(Θ) = 0.0031, the lowest in the compared literature, confirming both diagnostic confidence and honest uncertainty quantification.

Compared to DS-based competing methods, the proposed framework is strictly more general (DS is recovered as the special case $w_j = 1$), conflict-robust (it resolves the Zadeh paradox that DS cannot), source-aware (explicit reliability weights reflect differential diagnostic confidence of clinical parameters), and uncertainty-honest (m(Θ) rises with conflict rather than being suppressed).

Future work will extend the framework to larger multi-class clinical datasets, explore adaptive reliability weight estimation from historical diagnostic data, and investigate integration with machine learning models for automated membership function generation.

6. Declarations

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Competing Interests: The authors declare no conflicts of interest.

Data Availability: This study does not use actual patient data. The numerical illustration is based on secondary data adopted from Xiao [31] and is used solely for methodological validation. All data generated or examined in this study are contained within the article.

Ethics: There are no clinical trials, animal subjects, or human subjects in this study; ethical approval was not required.

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7. Nomenclature

All quantities are dimensionless mathematical or statistical measures (membership degrees, weights, entropies, and belief masses carry no physical SI units).

Symbol	Description	S.I. Unit
U	Universe of discourse (set of all elements x)	Dimensionless
$\mu(x)$	Membership function of a fuzzy set, $\mu: U \to [0,1]$	Dimensionless
A	A fuzzy set (or fuzzy soft set), $A = \{(x, \mu_a(x)) : x \in X\}$	Dimensionless
E	Parameter set of a soft/fuzzy soft set	Dimensionless
X	Universal set of alternatives in a fuzzy soft set	Dimensionless

F	Mapping from parameter set E to fuzzy subsets of $X, F : E \rightarrow P'(X)$	Dimensionless
d_{ij}	Membership value of alternative x_i under parameter e_j (fuzzy soft set matrix entry)	Dimensionless
$\hat{d}_{ij}$	Normalized membership value of d_{ij}(column-normalized so values sum to 1 for each parameter)	Dimensionless
t	Number of alternatives (diseases/hypotheses)	Dimensionless
n	Number of parameters (symptoms/evidence sources)	Dimensionless
$\|x_i\|$	Cardinality of element x_i	Dimensionless
$E_D(e_j)$	Deng entropy of parameter e_j	Dimensionless
$U(e_j)$	Exponential (zero-avoiding) uncertainty measure derived from Deng entropy of e_j	Dimensionless
$U'(e_j)$	Normalized uncertainty measure of parameter e_j(sums to 1 over all parameters)	Dimensionless
P	Fuzzy preference relation matrix, P	Dimensionless
p_{jk}	Degree of preference of alternative/parameter $e_j$over e_k, $p_{jk} \in [0,1]$	Dimensionless
$Var(e_j)$	Variance of the entropy values of all parameters other than e_j(degree of conflict of e_j)	Dimensionless
$Crd(e_j)$	Credibility value of parameter e_j	Dimensionless
$MU(e_j)$	Modulated uncertainty of parameter e_j (credibility-weighted uncertainty)	Dimensionless
$MU'(e_j)$	Normalized modulated uncertainty of parameter e_j	Dimensionless
w_j	Reliability weight assigned to parameter e_j, $w_j = 1 - MU'(e_j)$	Dimensionless
$m_j(x_i)$	Basic Probability Assignment (BPA) of disease x_i from parameter e_j	Dimensionless
θ	Frame of discernment (uncertainty/ignorance hypothesis)	Dimensionless
$m_j(\theta)$	BPA mass assigned to the frame of discernment θ (residual uncertainty) by parameter e_j	Dimensionless
m_1, m_2	Basic Probability Assignments of two independent evidence sources combined by the ER rule	Dimensionless
x_i	Candidate disease/hypothesis i (alternative in the frame of discernment Θ)	Dimensionless
Θ	Set of alternatives (hypotheses) $\{x_1, x_2, \ldots, x_n\}$	Dimensionless
I	Number of parameters (evidence sources) combined by the ER rule	Dimensionless
T_i	Intermediate product term in the ER combination rule for hypothesis x_i	Dimensionless
U (ER)	Aggregate uncertainty term in the ER combination rule, $UU = \prod_{j=1}^{I}(1 - w_j)$	Dimensionless

K	Normalization constant in the ER aggregation rule	Dimensionless
$m(x_i)$	Final normalized ER belief mass assigned to hypothesis x_i	Dimensionless
$m(\theta)$	Final normalized ER residual (ignorance) belief mass	Dimensionless
$Bel(x_i)$	Belief value (degree of support) for hypothesis x_i	Dimensionless

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